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The voting analytic hierarchy process method for discriminating among efficient decision making units in data envelopment analysis ☆

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ABSTRACT

Making optimal use of available resources has always been of interest to humankind, and different approaches have been used in an attempt to make maximum use of existing resources. Limitations of capital, manpower, energy, etc., have led managers to seek ways for optimally using such resources. In fact, being informed of the performance of the units under the supervision of a manager is the most important task with regard to making sensible decisions for managing them. Data envelopment analysis (DEA) suggests an appropriate method for evaluating the efficiency of homogeneous units with multiple inputs and multiple outputs. DEA models classify decision making units (DMUs) into efficient and inefficient ones. However, in most cases, managers and researchers are interested in ranking the units and selecting the best DMU. Various scientific models have been proposed by researchers for ranking DMUs. Each of these models has some weakness(es), which makes it difficult to select the appropriate ranking model. This paper presents a method for ranking efficient DMUs by the voting analytic hierarchy process (VAHP). The paper reviews some ranking models in DEA and discusses their strengths and weaknesses. Then, we provide the method for ranking efficient DMUs by VAHP. Finally we give an example to illustrate our approach and then the new method is employed to rank efficient units in a real world problem.

1. Introduction

Data envelopment analysis (DEA) was initiated in 1978 when Charnes, Cooper and Rhodes (CCR) demonstrated how to change a fractional linear measure of efficiency into a linear programming (LP) format (Charnes, Cooper, & Rhodes, 1978). DEA provides a relative efficiency measure for peer decision making units (DMUs) with multiple inputs and outputs. While DEA has been proven an effective approach in identifying the best practice frontiers, its flexibility in weighting multiple inputs and outputs and its nature of self-evaluation have been criticized. In most models of DEA, the best performers have the efficiency score unity, and from experience we know that there are usually plural DMUs that have this "efficient status". To discriminate among these efficient DMUs is an interesting research subject. Several authors have proposed methods for ranking the best performers. Andersen and Petersen (1993) observed that a DMU's efficiency possibly exceeds the conventional score 1.0 by comparing the DMU being evaluated with a linear combination of other DMUs while excluding the observations of the DMU being evaluated. They tried to discriminate among these efficient DMUs by using different efficiency scores larger than 1.0. Thrall (1996) pointed out that the model developed by Anderson and Peterson (AP) may result in instability when some inputs are close to zero. Then, to avoid this problem, MAJ (Mehrabian, Alirezaee, & Jahanshahloo, 1999) and SBM (Tone, 2002) models were proposed. Li, Jahanshahloo, and Khodabakhshi (2007) proposed a super-efficiency model that does not have the weakness in AP and MAJ models. Jahanshahloo, Pourkarimi, and Zarepisheh (2006) also showed that the technique used for rendering MAJ model unit invariant causes this model to give different rankings for two sets of DMUs that have identical conditions with respect to ranking. Then, they proposed a new technique which overcomes this problem (to a great extent). Moreover, to overcome the problem of instability in AP model, Sueyoshi (1999) use the modified slacks-based model to rank efficient units. Another approach known as the cross-evaluation method, proposed by Sexton, Silkman, and Hogan (1986), can be utilized as a DEA extension tool to identify best performing DMUs and to rank DMUs using cross-efficiency scores that are linked to all DMUs. Jahanshahloo, Junior, Hosseinzadeh Lotfi, and Akbarian (2007) proposed a new ranking system for extreme efficient DMUs based upon the omission of these efficient DMUs from the reference set of the inefficient DMUs. Liu and Peng (2008) introduced common weights analysis (CWA) to determine the single most favorable common set of weights for DMUs on the

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DEA frontier in view of maximizing the group's efficiency score. The assessment that proceeded based on the original DEA models shows that each DMU determines the efficiency score under its most favorable weights attached to its input indices and output indices.

In this paper, some ranking models in DEA are considered and their strengths and weaknesses are discussed. Finally, a method for ranking efficient units is proposed by combining the results of the above-mentioned models and the voting analytical hierarchy process (VAHP) approach. The remainder of the paper is organized into seven sections. Section 2 briefly introduces the background of DEA and VAHP. In Section 3, the strengths and weaknesses of some ranking models are addressed. We provide a background regarding voting models in Section 4 and present weights for ranking models based on their strengths, using a voting model. Section 5 proposes a method for ranking efficient DMUs with the VAHP approach. A numerical example is presented in Section 6 to illustrate our approach. In Section 7, an application of the new method to the performance measurement of bank branches in Iran is examined. Finally, the paper is concluded in Section 8.

2. Background

2.1. DEA background

DEA has evolved tremendously over the years and emerged as a body of concepts and methodologies, which consists of a collection of models and extensions to the original work of Charnes et al. (1978). In DEA, the organization under study is called a DMU. The definition of DMU is rather loose to allow flexibility in its use over a wide range of possible applications. Generically, a DMU is regarded as the entity responsible for converting inputs into outputs and whose performances are to be evaluated. As a performance measurement and analysis technique, DEA is a non-parametric frontier estimation methodology based on linear programming for evaluating the relative efficiency of a set of comparable DMUs that share common functional goals.

Suppose an organization has n DMUs ($DMU_j, j = 1, 2, \ldots, n$), produces s outputs denoted by y_{rj}, the rth output of DMU_j for $r = 1, 2, \ldots, s$, and consumes m inputs denoted by x_{ij}, the ith input of DMU_j for $i = 1, 2, \ldots, m$. The following LP models are the conventional CCR models for efficiency analysis. Let the DMU being evaluated on any trial be designated as DMU_o, where o ranges over $1, 2, \ldots, n$. We can solve the following fractional programming problem (1) or linear programming problem (2) to obtain the objective value (relative efficiency θ_o^*) and one comparative set of weights of inputs (v_{io}, $i = 1, 2, \ldots, m$) and outputs (u_{ro}, $r = 1, 2, \ldots, s$). ε is a positive Archimedean infinitesimal constant, which is used in order to avoid the appearance of zero weights. This zero case in weights would result in the meaninglessness of certain indices used in DEA.

$$\begin{aligned} \theta_o^* &= \max \frac{\sum_{r=1}^{S} u_{ro} y_{ro}}{\sum_{i=1}^{m} v_{io} x_{io}} \\ \text{s.t. } & \frac{\sum_{r=1}^{S} u_{ro} y_{rj}}{\sum_{i=1}^{m} v_{io} x_{ij}} \leqslant 1, \quad j = 1, \ldots, n \\ u_{ro} &\geqslant \varepsilon > 0, \quad r = 1, \ldots, s \\ v_{io} &\geqslant \varepsilon > 0, \quad i = 1, \ldots, m \end{aligned} \tag{1}$$

$$\begin{aligned} \theta_o^* &= \max \sum_{r=1}^{s} u_{ro} y_{ro} \\ \text{s.t. } & \sum_{i=1}^{m} v_{io} x_{io} = 1 \\ & \sum_{r=1}^{s} u_{ro} y_{rj} - \sum_{i=1}^{m} v_{io} x_{ij} \leqslant 0, \quad j = 1, \ldots, n \\ u_{ro} &\geqslant \varepsilon > 0, \quad r = 1, \ldots, s \\ v_{io} &\geqslant \varepsilon > 0, \quad i = 1, \ldots, m \end{aligned} \tag{2}$$

It is claimed that DMU_o is comparatively efficient (also called an efficient DMU) with the efficiency $\theta_o^* = 1$. We define $E = \{j | \theta_j^* = 1, j = 1, 2, \ldots, n\}$ to represent the set of efficient DMUs. It is helpful for decision makers only to focus on the efficient DMUs. However, decision makers always face the problem of how to carry out a further comparison among DMUs in set E.

2.2. VAHP background

The analytical hierarchy process (AHP), introduced by Saaty (1980) based on pairwise comparisons, has been applied to alternative selection. Narasimhan (1983), Partovi and Banerjee (1989), Nydick and Hill (1992), Barbarosoglu and Yazgac (1997), Yahya and Kingsman (1999), Masella and Rangone (2000), Tam and Tummala (2001), and Lee, Ha, and Kim (2001) proposed to use this technique to cope with determining scores. Also, Liu and Hai (2005) presented the voting analytical hierarchy process (VAHP), as a novel easier weighing procedure in place of AHP's paired comparison. The VAHP approaches AHP, it allows the purchasing manager to generate non-inferior purchasing options and systematically analyze the inherent trade-offs among the relevant criteria. Liu and Hai (2005) discussed so far the applicability of the ranking method initiated by Noguchi, Ogawa, and Ishii (2002) and, by using DEA, they determined the weights from rank voting data. Then, they showed that the total ordinal rank of objects may produce a different result according to the difference of the weights between ranks. Finally, they extended these ordering methods to multi-purpose evaluation, e.g., employee selection, appraising performance of individual or departments, etc. They summarized their method in six stages. In summary, comparing the benefits of the VAHP to AHP, they find that:

1. The VAHP method is simple to understand and use for obtaining priority weights. All experts were given the opportunity to examine the priority weights calculated from their initial responses and to assess the reasonableness of the ranking. When their results seemed counterintuitive, they were encouraged to reevaluate their input data, determine the source of the inconsistency, and make the appropriate changes.
2. The construction of the objective hierarchy of criteria, attributes and alternatives facilitates communication of the problem and solution recommendation.
3. It provides "vote ranking" rather than "paired comparison" quantifying and measuring consistence.
4. The paired comparison used to weight the criteria in the AHP is more difficult than the vote ranking which is used in the VAHP.
5. The strongest feature of the AHP is its generation of numerical priorities from the subjective knowledge expressed in the estimates of paired comparison matrices.

They used the vote ranking to determine the weights in the selected rank, in place of the paired comparison method. In the six step procedure, the difference between VAHP and AHP lies in steps 3 and 4 in Table 1 (taken from Liu and Hai (2005)).

3. Strengths and weaknesses of ranking models

In this section, we discuss the strengths and weaknesses of ranking models and state the strengths of some existing ranking models. The ranking models presented so far by different authors have strengths and weaknesses. For instance, the cross-efficiency model encounters the issue of selecting the best solution when the problem has multiple optimal solutions. AP model gets infeasible for some DMUs and is unstable. Most existing models are only able to rank extreme efficient DMUs, but not non-extreme efficient ones. Therefore, no single model can be specified as the best ranking model whose results can be relied on in all cases. However, weights can