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Optimal maintenance time for imperfect maintenance actions on repairable product ☆

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ABSTRACT

This paper develops a maintenance strategy for repairable products that combines imperfect maintenance actions at pre-scheduled times and minimal repair actions for failures. Under a power law process of failures, an expected total cost is developed that involves the sum of the total cost of imperfect preventive maintenances and the expected total cost of minimal repairs. Moreover, a searching procedure is provided to determine the optimal maintenance schedule within a finite time span of warranty. When the parameters of the power law process are unknown, the accuracy of the estimated maintenance schedule is evaluated based on data through an asymptotic upper bound for the difference of the true expected total cost and its estimate. The proposed method is applied to an example regarding the maintenance of power transformers and the performance of the proposed method is investigated through a numerical study. Numerical results show that the proposed maintenance strategy could save cost whether an imperfect maintenance action or the perfect maintenance action is implemented.

1. Introduction

Most products or systems are designed to be repaired rather than replaced after failure in the real world. Maintenance policies are fundamental under these conditions because a properly preventive maintenance (PM) strategy can save money and keep products running longer. A PM policy specifies the periodicity to maintain a product through the product whole lifetime. Pham and Wang (1996) mentioned that a maintenance action could be classified into perfect maintenance, minimal repair (MR) or imperfect maintenance. A perfect maintenance restores a product to be as good as new, an MR restores a product to have the same failure rate condition as it had just right before failure and an imperfect maintenance makes a product better than what it had before failure but not necessarily to be as good as new. Since the pioneer work of Barlow and Hunter (1960), the combination of a perfect PM and an MR has been of interest by many authors, for examples, Gerstack (1977), Block, Borges, and Savits (1990), Park, Jung, and Yum (2000), Lai, Leung, Tao, and Wang (2001) and Gilardoni and Colosimo (2007).

In an optimal maintenance policy setting, the nonhomogeneous Poisson process (NHPP) has played a key role in modeling the random occurrences of failures. Let $N(0,t)$ denote the number of failures in the interval $(0,t]$. A process $\{N(0,t):t \geqslant 0\}$, which has independent increments and $N(0,0) = 0$, is a Poisson process with intensity $\lambda(t)$, if the random variable $N(0,t)$ has a Poisson distribution and mean $M(t) = E(N(0,t)) = \int_0^t \lambda(u)du$ for $t \geqslant 0$. When the intensity function $\lambda(t)$ is not constant and depends on the time t, the Poisson process is called the NHPP. The most popular NHPP is the power law process (PLP) which has a Weibull intensity function,

$$\lambda(t) = \beta t^{\beta-1}/\theta^{\beta}, \tag{1.1}$$

where $\theta > 0$ is the scale parameter and $\beta > 0$ is the shape parameter.

The PLP had been successfully applied to model the occurrences of failures in a number of PM studies. Some good discussions regarding the applications of NHPP have been published by Crow (1974), Cox and Miller (1965), Ascher and Feingold (1984), Bain and Engelhardt (1991), Rigdon and Basu (2000) and Pulcini (2001). The model (1.1) is quite flexible in reliability studies because it includes the growth model when $0 < \beta < 1$, the decay model when $\beta > 1$ and the homogeneous Poisson process when $\beta = 1$.

Assuming infinite operation time for a repairable product, Gilardoni and Colosimo (2007) proposed an optimal perfect PM schedule which minimized the expected average total cost per unit time. Moreover, they provided a large sample estimation procedure for the determination of their PM schedule when the parameters of the PLP are unknown. However, the maintenance actions in practical situations could be imperfect and the operation time for a repairable product could be finite. This article relaxes the

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conditions of Gilardoni and Colosimo (2007) to develop a new PM plan in which repairable products undergo imperfect maintenance actions within a finite time span of warranty. The objective of the proposed PM policy is to minimize the expected total cost in a finite time span of warranty instead.

In Section 2, the proposed PM policy and a searching procedure to setup the optimal PM schedule are developed for repairable products when the time span of warranty is finite. In Section 3, an asymptotic upper bound for the difference of the true expected total cost and its estimate is provided to evaluate the accuracy of the estimated PM schedule based on data. The proposed method is illustrated via an example in Section 4. Moreover, the performance of the proposed PM policy is compared with the one proposed by Gilardoni and Colosimo (2007) in terms of the expected average total cost per unit time. In Section 5, a numerical study is conducted to evaluate the performance of the proposed PM policy for various combinations of parameters. Finally, concluding remarks are given in Section 6.

2. The proposed preventive maintenance model

Assume that a repairable product starts to operate at time zero and undergoes m times of imperfect maintenance actions within a finite time span of warranty, W. The imperfect maintenance action satisfies the following conditions:

1. PM check points are scheduled after every τ units of time such that $0 \leqslant m\tau \leqslant W$.
2. Each PM can return the product's age $x_i = x = r\tau$, where $i = 1,2,\ldots,m$ and $0 \leqslant r \leqslant 1$. If $r = 0$, then the maintenance action has no effect to the product; while $r = 1$, it represents a perfect maintenance action which instantly returns the product to a new condition.
3. The PM cost at the time t can be modeled as a linear function of the product's age t and the returned product's age x, $C_p(t,x) = a + c_1x + c_2t$, where a, c_1 and c_2 are nonnegative coefficients (see Yeh & Chen (2005)).
4. When a failure occurs between two PM check points, a MR is applied. The cost for a MR is denoted by c_{MR}.

It should be noticed that if no PM action is implemented in the operating time interval $(0,W]$, then the expected total cost is given as

$$c_0 = c_{MR} \cdot E[N(0,W)]. \tag{2.1}$$

Otherwise, let the time interval $((0,W] = (0,\tau] \cup (\tau,2\tau] \cup \cdots \cup ((m-1)\tau, m\tau] \cup (m\tau, W]$ and y_i denote the i^{th} cumulative return time at the i^{th} PM action, where $i = 0,1,2,\ldots,m$, $y_0 = 0$ and $y_i = \sum_{k=1}^{i} x_i = ir\tau$ for $i = 1,2,\ldots,m$. Therefore, the expected MR cost in the i^{th} interval $((i-1)\tau, i\tau]$ is the c_{MR} multiple of the expected number of failures that occur within the interval. Moreover, the expected cost in the ith interval $((i-1)\tau, i\tau]$ is the sum of the expected MR cost in the interval and the PM cost happened at the end of the interval. Therefore, the expected cost in the ith interval can be mathematically represented as,

$$c_{MR} \cdot E[N((i-1)\tau - y_{i-1}, i\tau - y_{i-1})] + C_p(i\tau, x). \tag{2.2}$$

The expected total cost for the entire time interval, $(0,W]$ can be determined as follows:

$$C(\tau,m) = \sum_{i=1}^{m} \{c_{MR} \cdot E[N((i-1)\tau - y_{i-1}, i\tau - y_{i-1})] + C_p(i\tau, x)\} + c_{MR} \cdot E[N(m\tau - y_m, W - y_m)], \tag{2.3}$$

where $E[N(h_1,h_2)] = \int_{h_1}^{h_2} \lambda(u)du$, $h_1 < h_2$. When a perfect PM is applied and $c_1 = c_2 = 0$, $C(\tau,m)$ is reduced to Eq. (1) of Gilardoni & Colosimo (2007) with $T = W$. However, when $T = W$ is finite, the term $R = c_{MR} \cdot E[N(m\tau - y_m, W - y_m)]$ may not be negligible and the average expected total cost per unit of time $\frac{C(\tau,m)}{W}$ is, hence, different from the Eq. (2) of Gilardoni & Colosimo (2007) under a perfect PM with $c_1 = c_2 = 0$. The derivative of $C(\tau,m)$ with respect to τ is

$$\frac{dC(\tau,m)}{d\tau} = \sum_{i=1}^{m} \{c_{MR} \cdot [a_i\lambda(a_i\tau) - b_i\lambda(b_i\tau)] + (c_1r + c_2i)\} - c_{MR} \cdot [mr\lambda(W - mr\tau) + m(1-r)\lambda(m(1-r)\tau)], \tag{2.4}$$

where $a_i = i - ir + r$ and $b_i = (i-1)(1-r)$. Under the PLP of the Weibull intensity function, Eq. (2.4) can be rewritten as,

$$\frac{dC(\tau,m)}{d\tau} = \sum_{i=1}^{m} \left[\frac{c_{MR}\beta\tau^{\beta-1}}{\theta^\beta}\left(a_i^\beta - b_i^\beta\right) + c_1r + c_2i\right] - \frac{c_{MR}}{\theta^\beta}\left[\beta mr(W - mr\tau)^{\beta-1} + (m(1-r))^\beta \beta\tau^{\beta-1}\right] \tag{2.5}$$

and the second derivative of $C(\tau,m)$ with respect to τ is

$$\frac{d^2C(\tau,m)}{d\tau^2} = \left(\frac{c_{MR}\beta(\beta-1)}{\theta^\beta}\right)\left\{\tau^{\beta-2}\left[\sum_{i=1}^{m}\left(a_i^\beta - b_i^\beta\right) - (m(1-r))^\beta\right] + (mr)^2(W - mr\tau)^{\beta-2}\right\}. \tag{2.6}$$

It can be shown that $\sum_{i=1}^{m}\left(a_i^\beta - b_i^\beta\right) - (m(1-r))^\beta > 0$ for $\beta > 0$. Therefore, when $\beta > 1$, $\frac{d^2C(\tau,m)}{d\tau^2} > 0$ and $C(\tau,m)$ is a convex function of τ over $0 < \tau \leqslant W$ for a given m. Because the repairable product is assumed to decay in reliability, only the case of $\beta > 1$ is considered. Hence, the optimal τ_m^*, which minimizes the expected total cost in the time interval $(0,W]$ can be determined by solving $dC(\tau,m)/d\tau = 0$ over $0 < \tau < W$ or $\tau_m^* = W$. For each m, the optimal τ_m^* can be solved numerically. The optimal PM number m^* can be obtained by

$$m^* = \arg\min_{\{m=1,2,\ldots\}} \{C(\tau_m^*, m)\}.$$

In practical applications, it is common for the MR cost to greatly exceed the PM cost. In addition, the expected number of failures is increasing with respect to the length of operating time interval between two PMs when the PLP shape parameter $\beta > 1$ and each PM can improve the system state. Therefore, when the number of PMs starts to increase from zero, it is expected to decrease the expected total cost, intuitively. However, when the number of PMs increases to a certain level, the expected total cost would start to increase. Based on this principle, a search algorithm shown in Fig. 1 is proposed to find the optimal PM time schedule.

3. Statistical methods

In practical applications, parameters in the NHPP may not be known in advance. It is necessary to estimate τ based on data. Assuming that a product could be operated for infinite time, Gilardoni & Colosimo (2007) discussed a procedure for the large sample maximum likelihood estimation of their optimal maintenance schedule based on the failure times observed from one or more identical products under their proposed perfect PM policy. The basic ways for collecting data from a repairable product could be the failure truncated or the time-truncated sampling. The failure truncated sampling means that the data collection is ceased after a specified number, k, of failures. The time truncated sampling means that the data collection is ceased at a predetermined time T.

Let $0 < t_1 < t_2 < \cdots < t_k < T$ denote the times to failures observed until a predetermined time T for a NHPP with intensity function $\lambda(t) = \lambda(t|\boldsymbol{\mu})$, where $\boldsymbol{\mu}$ is a vector of the unknown parameters.