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Two general forms of approximation for determining near optimal inspection intervals with non-zero inspection and replacement times in a deteriorating production system

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Abstract

The construction and solution of the next simplest but more realistic profit model for an exponential shift (or failure) distribution were presented in Ben-Daya and Hariga (B&H) [Ben-Daya, M., & Hariga, M. (1998). A maintenance inspection model: Optimal and heuristic solutions. *International Journal of Quality and Reliability Management, 15*, 481–488], in which the model presented looks much more complicated than the simplest profit model proposed by Baker [Baker, M. J. C. (1990). How often should a machine be inspected? *International Journal of Quality and Reliability Management, 7*, 14–18]. However, once we confer economic meanings to some mathematical expressions and rewrite the whole model, the revised model not only becomes less complicated and more comprehensible than the original one but also draws a closer analogy with Baker's model and the associated solution algorithms.

We also develop two distinct sets of general explicit formulae and present two corresponding heuristic algorithms for solving B&H's model. The examples show that either of these new algorithms for determining a near optimal inspection interval for a deteriorating (or an imperfect) production system is not only more close to the optimum but also more efficient computationally than the one suggested by B&H (1998), and can give the value to any required degree of accuracy.

The main purpose of this article is threefold: (1) to confer economic meanings to some expressions, (2) to rewrite the model and the Proof of Theorem 1, and (3) to introduce general exponential and logarithmic forms of approximation for further approximating optimal inspection intervals and to give merits and demerits of the two algorithms.

Keywords: Inspection; Replacement; Cost; Profit; Machine; Shift

1. Introduction

Consider the production of a single item on a single unit representing a production system composed of many components. In the following, the author will use the word "machine" to refer to such a single unit

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or complex system. It is assumed that, at the start of the production cycle, the machine is in the "in-control" state, producing items of perfect quality. However, after a production time period, the machine may shift to an "out-of-control" state. From that point on, it is assumed that all the items produced are sub-standard but still of acceptable quality. In other words, production quality is not always perfect but is usually dependent on the machine's state. The time that elapses while the machine is in the in-control state, before the shift occurs, is a random variable Y, assumed to follow an exponential distribution with a constant hazard λ per unit time, i.e., $F(y) = 1 - e^{-\lambda y}$ for $\lambda > 0$ and $y \geqslant 0$. Once a shift to the out-of-control state has occurred, we assume that the machine stays in that state unless it is discovered by inspection (i.e., a shift is not apparent as the machine carries on producing items of unremarkably inferior quality) and followed by some restorative work. Notice that a continuous monitoring of operating states is not economically justifiable for some machines. Alternatively, inspections are useful in monitoring the machine's condition or the items' quality, and these can be made periodically at fixed multiples of some predetermined time interval T in order to reduce the probability of shift. Frequent inspection increases inspection costs while infrequent inspection leads to reducing profits (attributed to the triple effect that an out-of-control state could correspond to an increased production cost per unit time in addition to reduced selling prices of inferior items; also the later we realize that the machine is out-of-control, the more costly it would be to restore it). Thus, an economically optimal inspection interval usually exists.

The above distinction between in-control and out-of-control states, and the restrictive assumption of an exponential distribution for the elapsed time of the in-control state are commonly found in the economic production quantity (EPQ) literature such as Lee and Rosenblatt (1987) or the quality control literature such as Juran and Gryna (1980). Historically, it has been found that many components and systems experience exponential failures (regarded as out-of-control states). Davis (1952) found that this failure law characterizes a wide variety of devices including ball-and-roller bearings, vacuum tubes, bus engines, and many electronic systems. Drenick (1960) mathematically showed that under reasonably general conditions, distribution of the time between failures tends to the exponential as the machine structure complexity or the operation time increases. Throughout this article, the paired terminologies: "as-good-as new state" and "in-control state", "perfect repair" and "replacement", "failure" and "shift" or "failed state" and "out-of-control state" are equivalent.

Many authors in the maintenance literature have considered different variations of this single machine inspection problem. Kamins (1960) and Coleman and Abrams (1962) extensively studied inspection procedures to maximize availability. Each considered the possibility that inspection might subject the machine to further stresses that might lead to failure. The probabilities of calling a good machine bad and a bad machine good were included in their analysis. The main difference between these two studies is that the former used the constant time T while the latter used the expected time between two successive inspections in determining machine availability. Other distinguished studies are for example, Jacobs (1968), Vaurio (1979), Voelker and Pierskalla (1980), McWilliams and Martz (1980), Sim (1985) and Banerjee and Chuiv (1996). A rather detailed literature review on inspection-scheduling problems proposed by Barlow et al. (1963) and Brender (1963) can be found in Leung (2001). Recent articles concerning inspection problems are Yang and Klutke (2001), Lam (2003), Cui et al. (2004), and Zequeira and Berenguer (2006).

Baker (1990) proposed the simplest model, based on the restrictive assumption that a failure completely halts production, for finding the optimal inspection frequency that generates maximum profit. Chung (1993), Vaurio (1994), and Hariga (1996) subsequently developed solution algorithms for Baker's model. Assuming that a perfect maintenance policy is followed instead of performing just an inspection at the end of each cycle, Hariga (1996) generalized Baker's model by allowing the failure time to follow a general type of distribution such as Weibull.

Under the exponential shifting time from the in-control state where items of perfect quality are produced to an out-of-control state where items of sub-standard quality are produced, Ben-Daya and Hariga (1998) (hereafter referred to as B&H) and Hariga and Al-Fawzan (2000, chap. 10) reformulated Baker's model, respectively, by incorporating constant inspection and replacement times and by using the concept of discounted cash flow analysis to account for the effects of the time value of money on inspection policies. The joint problem of maintenance inspection schedules and EPQ has been studied by many researchers, see e.g., Lee and Rosenblatt (1987), where they determined the optimal length of the production cycle and the number of