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Optimal replenishment policy for perishable items with stock-dependent selling rate and capacity constraint ☆

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ABSTRACT

In this paper, a deterministic inventory model is developed for deteriorating items with stock-dependent demand and finite shelf/display space. Furthermore, we allow for shortages and the unsatisfied demand is partially backlogged at the exponential rate with respect to the waiting time. We provide solution procedures for finding the maximum total profit per unit time. In a specific circumstance, the model will reduce to the case with no shortage. Further, we use numerical examples to illustrate the model.

1. Introduction

In the last several years, the influence of displayed stock level on customers has been recognized by many marketing researchers and practitioners. High inventories might stimulate demand for a variety of reasons. For example, tall stacks of a product can promote visibility, thus kindling latent demand. A large inventory might also signal a popular product, or provide consumers an assurance of high service levels and future availability. Having many units of a product on hand also permits a retailer to disperse the product across multiple locations on the sales floor, thereby potentially capturing additional demand. Researchers like Levin, McLaughlin, Lamone, and Kottas (1972) and Silver and Peterson (1985) observed the functional relationship between the demand and the on-display stock level. Due to this fact, various functional forms of the stock-dependent demand rate were assumed to analyze inventory control policies under realistic situations. Baker and Urban (1988) considered a power-form inventory-level-dependent demand rate, which would decline along with the stock level throughout the entire cycle. Datta and Pal (1990) modified the model of Baker and Urban (1988) by assuming that the stock-dependent demand rate was down to a given level of inventory, beyond which it is a constant. Gupta and Vrat (1986) assumed that the demand rate was a function of initial stock level. Mandal and Phaujdar (1989) then developed on deteriorating inventory model in the case of deterministic demand rate that depended linearly on the instantaneous stock level.

Due to various uncertainties, the occurrence of shortages in inventory is a natural phenomenon. Hence, Urban (1995) extended Baker and Urban's (1988) model to allow shortages, in which unsatisfied demand is backlogged at a fixed fraction of the constant demand rate. Padmanabhan and Vrat (1995) developed an inventory model in which the backlogging rate depends upon the total number of customers in the waiting line (i.e., the amount of the negative inventory level). Therefore, the more the amount of demand backlogged, the smaller the demand to accept backlogging would be. Their definition of backlogging rate, however, seems to be inappropriate when customers do not know how many buyers waiting before him/her. When there is a shortage, most customers only concern on the duration he/she has to wait.

During the shortages period, often some customers are conditioned to a shipping delay, and may be willing to wait for a short time, while other will leave for another seller because of urgent need. Therefore, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not. To reflect this phenomenon, Abad (1996), Abad (2001), Abad (2008) discussed a pricing and lot-sizing problem for a product with a variable rate of deterioration, allowing shortages and partial backlogging. The backlogging rate depends on the time to replenishment – the longer customers must wait, the greater the fraction of lost sales. Since Abad (1996) proposed two specific examples of impatient functions – the exponential rate and the hyperbolic rate with respect to waiting time, these

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two functions have been used to model backordering in several studies, for example, Papachristos and Skouri (2000), Teng, Chang, Dye, and Hung (2002), Skouri and Papachristos (2003), San Jose, Sicilia, and Garcia-Laguna (2006) and Dye (2007).

Companies have recognized that besides maximizing profit, customer satisfaction plays an important role for getting and keeping a successful position in a competitive market. Common measures for customer service in inventory literature are the non-stockout probability per replenishment cycle. With a lost sale, the customer's demand for the item is lost and presumably filled by a competitor. It can be considered as the loss of profit on the sales. Ignoring the stockout cost from the total profit leads to the overrated profit and less customer satisfaction. Therefore, information about the backlogging rate during the waiting time is required and it is of considerable interest to understand the trade-off between the investment in inventory and the service level. Recently, Dye and Ouyang (2005) amended Padmanabhan and Vrat (1995) partial backlogging model by adding the cost of lost sales in the profit function and using Abad's linear time-proportional backlogging rate. They established a unique optimal solution to the problem in which building up inventory has a negative effect on the profit. Later, Chang, Goyal, and Teng (2006) complemented Dye and Ouyang's model for the situation that building up inventory has a positive impact on the profit by limiting the capacity of shelf space. Wu, Ouyang, and Yang (2006) considered the problem for non-instantaneous deteriorating items with stock-dependent demand to determine the lot size at the hyperbolic backlogging rate. However, the exponential backlogging rate has not been used to model the inventory problem for deteriorating items with stock-dependent demand.

Hence, the main purpose of this paper is to determine the optimal replenishment policy for deteriorating items with stock-dependent demand. In the model, shortages are allowed, and the rate of backlogged demand decreases exponentially as the waiting time for the next replenishment increases. In addition, the capacity of shelf space is assumed to be finite. The rest of the paper is organized as follows. In the next section, the notation and assumptions related to this study are presented. In Section 3, we develop the criterion for finding the optimal solution for the replenishment schedule, and prove that the optimal replenishment policy not only exists but is unique. In the last two sections, numerical examples are used to illustrate the procedure of solving the model and concluding remarks are provided.

2. Notation and assumptions

2.1. Notation

To develop the mathematical model of inventory replenishment schedule, the notation adopted in this paper is as below:

A	the replenishment cost per order
c	the purchasing cost per unit
s	the selling price per unit, where $s > c$
Q	the ordering quantity per cycle
B	the maximum inventory level per cycle, i.e., the initial inventory level.
i	the carrying cost rate, the cost of having one dollar of the item tied up in inventory per unit time
c_2	the shortage cost per unit per unit time
c_3	the cost of lost sales (i.e., goodwill cost) per unit
t_1	the length of period during which the inventory level reaches zero, where $t_1 \geqslant 0$
t_2	the length of period during which shortages are allowed, where $t_2 \geqslant 0$
T	the length of the inventory cycle, hence $T = t_1 + t_2$
$I_1(t)$	the level of positive inventory at time t, where $0 \leqslant t \leqslant t_1$
$I_2(t)$	the level of negative inventory at time t, where $t_1 \leqslant t \leqslant t_1 + t_2$
$TP(t_1, t_2)$	the total profit per unit time

2.2. Assumptions

In addition, the following assumptions are imposed:

1. Replenishment rate is infinite, and lead time is zero.
2. The time horizon of the inventory system is infinite.
3. The demand rate function, $D(t)$, is deterministic and is a function of instantaneous stock level $I(t)$; the functional $D(t)$ is given by:
$$D(t) = \begin{cases} \alpha + \beta I(t), & 0 \leqslant t \leqslant t_1 \\ \alpha, & t_1 \leqslant t \leqslant t_1 + t_2 \end{cases}$$
where α and β are non-negative constants.
4. The distribution of time to deterioration of the items follows exponential distribution with parameter θ (i.e. constant rate of deterioration).
5. The retail outlets have limited shelf space. It is W units, where $W \geqslant B$.
6. Shortages are allowed. We adopt the concept used in Abad (1996), Abad (2001), i.e., the unsatisfied demand is backlogged, and the fraction of shortages backordered is $e^{-\delta x}$, where x is the waiting time up to the next replenishment and δ is a positive constant.

3. Mathematical formulation

Using above assumptions, the inventory level follows the pattern depicted in Fig. 1. To establish the total profit function, we consider the following time intervals separately, $[0, t_1]$ and $[t_1, t_1 + t_2]$. During the interval $[0, t_1]$, the inventory is depleted due to the combined effects of demand and deterioration. Hence the inventory level is governed by the following differential equation:

$$\frac{dI_1(t)}{dt} = -\alpha - \beta I_1(t) - \theta I_1(t), \quad 0 < t < t_1, \tag{1}$$

with the boundary condition $I_1(t_1) = 0$. Solving the differential Eq. (1), we get the inventory level as

$$I_1(t) = \frac{\alpha}{\beta + \theta}[e^{(\beta+\theta)(t_1 - t)} - 1], \quad 0 \leqslant t \leqslant t_1. \tag{2}$$

Hence the maximum inventory level per cycle is

$$B = I_1(0) = \frac{\alpha}{\beta + \theta}[e^{(\beta+\theta)t_1} - 1]. \tag{3}$$

Furthermore, at time t_1, shortage occurs and the inventory level starts dropping below 0. During $[t_1, t_1 + t_2]$, the inventory level only depends on demand, and a fraction $e^{-\delta(t_1+t_2-t)}$ of the demand is backlogged, where $t \in [t_1, t_1 + t_2]$. The inventory level is governed by the following differential equation:

$$\frac{dI_2(t)}{dt} = -\alpha e^{-\delta(t_1+t_2-t)}, \quad t_1 < t < t_1 + t_2, \tag{4}$$

with the boundary condition $I_2(t_1) = 0$. Solving the differential Eq. (4), we obtain the inventory level as

$$I_2(t) = -\frac{\alpha}{\delta}[e^{-\delta(t_1+t_2-t)} - e^{-\delta t_2}], \quad t_1 \leqslant t \leqslant t_1 + t_2. \tag{5}$$