Computers & Industrial Engineering 62 (2012) 1011–1024

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Three m-failure group maintenance models for $M/M/N$ unreliable queuing service systems

Gia-Shie Liu *

Department of Information Management, Lunghwa University of Science and Technology, No. 300, Sec. 1, Wanshou Rd., Guishan Shiang, Taoyuan County 33306, Taiwan

ARTICLE INFO

Article history:
Received 15 September 2009
Received in revised form 6 November 2011
Accepted 19 December 2011
Available online 5 January 2012

Keywords:
Group maintenance
m-Failure
Matrix-geometric
Queuing system
Continuous time Quasi Birth-and-Death Processes

ABSTRACT

This paper considers group maintenance problems for an unreliable service system with N independent operating servers and a Markovian queue. A specific class of group maintenance policies is developed where the repair is started as soon as the number of failed servers reaches a predetermined threshold. This is actually a Quasi Birth-and-Death Process with two dimensions, the level for the arrival/service process and the phase for the failure/repair process. Two models with positive repair time and another with instantaneous repair are considered. The matrix geometric approach is applied to calculate the steady state distribution and the expected average cost for all three models. For the theoretical analysis, this paper proves that there exists an optimal group maintenance parameter m^*, which can find the minimal average cost for all three models. Additionally, some mathematical properties and sensitivity analyses are numerically demonstrated based on various parameters. Finally, the comparisons of these three proposed models in many aspects are also discussed.

1. Introduction

It is very important to keep service systems operating normally in this service oriented era. Falling this, a lot of cost would occur due to the loss of customers and the delay of the production of goods. For examples, most telephone companies or mobile phone companies will lose their customers because of frequent communication traffic jam problems caused by failures of their service systems; the failure of the related power systems operated by the power company will cause the production delay of some manufacturing companies; moreover, for some systems such as nuclear power systems, military weapon systems, airplanes, and submarines, it is exceptionally essential to keep away from breakdowns during their operation since it can be falled in a very dangerous and disastrous condition. To reach this goal, besides inherent reliability, an appropriate maintenance policy can be applied to avoid failures of such operating systems.

This paper will focus on maintenance problems of some specific large service or operating systems such as telecommunication systems, internet service systems, electric power systems, nuclear power systems, military submarines, aircraft carriers, space stations, satellite systems, and automated manufacturing systems. Since the services or functions provided by these systems are so important, most of them are multi-unit systems, which can apply parallel or redundant system designs to keep the whole system operating normally even some service units are malfunctioning. Of course, the multi-unit system can also provide more sufficient services to meet all kinds of requests or missions given by customers. The simplest maintenance policy for the multi-unit system is to treat all units independently and follow the same maintenance policy separately for each unit in the system. Nevertheless, for those specific systems focused by this paper, the replacement activity is not possible or difficult to be implemented immediately upon every single unit failure because the missions or operations cannot be canceled immediately for safety or it will cause a lot of setup cost, production delaying cost, customer holding cost or customer loss cost to stop the service/production operations. In that case, it is obvious that group maintenance policies can play a key role to keep those multi-unit systems operating normally.

In recent years, a large amount of researches has been devoted to finding optimal maintenance policies under various assumptions. The maintenance models for multi-unit systems are generally based on those for single-unit systems. For single-unit systems, various maintenance policies, such as block replacement policies (Beichelt, 1993; Chien & Chen, 2007; Kennee, Gharbi, & Beit, 2007; Scarf, Dwight, & Al-Musrati, 2005; Sheu, 1997, 1998; Sheu & Griffith, 2002), age-replacement policies (Beichelt, 1993; Scarf et al., 2005; Sheu, 1998; Shen & Chien, 2004), periodic preventive maintenance policies (Jung & Park, 2003; Jung, Park, & Park, 2010; Sheu, Lin, & Liao, 2006; Vaughan, 2005; Yeh & Lo, 2001), failure limit maintenance policies (Arunraj & Maiti, 2010; Carazas & Souza, 2010; Chan & Asgarpoor, 2006; Das, Lashkari, & Sengupta, 2007; Grall, Berenguer, & Dieulle, 2002; Lapa, Pereira,

* Tel.: +886 2 26788221 (H), +886 2 82093211x6328 (O); fax: +886 2 26788221.
E-mail addresses: liugtw@yahoo.com.tw, liug@mail.lhu.edu.tw

0360-8352/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cie.2011.12.028

Nomenclature

c	number of repairmen
e	$[1,1,\ldots,1]'$
f	failure rate for each server
m	the threshold of number of failed servers to initiate repair process
m^*	the optimal m
N	total number of servers in the system
n	the repaired number of failed servers
$\underline{Q}$	infinitesimal generator for failure/repair process
$\overline{Q}$	infinitesimal generator for $M/M/N$ continuous time Markov Process
$\overline{P}$	the related transition probability matrix derived from $\overline{Q}$
r	repair rate with one repairman fixing one failed server
r_w	repair rate when w servers are operational
T	the threshold of scheduled time to initiate repair process
τ	another threshold of scheduled time to initiate repair process
T^*	the optimal T
w	number of operational servers in the system
x	number of customers in the system
y_{xw}	probability of x customers in system and w servers are operational
y_x	$[y_{x0}, y_{x1}, \ldots, y_{xN}]$
$\bar{y}$	$[y_0, y_1, y_2, \ldots]$
λ	customer arrival rate
λ_w	customer arrival rate when w servers are operational
$\bar{\lambda}$	vector of arrival rate in each state of number of operating servers
$\Delta(\lambda)$	$diag\ (\lambda_0, \lambda_1, \ldots, \lambda_N)$
μ	service rate of one server for each customer
μ_w	service rate when w servers are operational
$\bar{\mu}$	vector of service rate in each state of number of operating servers
$\Delta(\bar{\mu})$	$diag\ (0, \mu, 2\mu, \ldots, N\mu)$
$\Delta(\bar{\lambda}+\bar{\mu})$	$diag\ (\lambda_0, \lambda_1+\mu, \lambda_2+2\mu, \ldots, \lambda_N+N\mu)$
π_w	the stationary probability of w servers are operational
$\bar{\pi}$	$[\pi_0, \pi_1, \ldots, \pi_N]$
S	fixed repair cost
r_cost	variable repair cost per failed server
h	holding cost per customer per unit time
A_0	$\Delta(\bar{\lambda})$
A_2	$\Delta(\bar{\mu})$
A_2	$Q-\Delta(\bar{\lambda})-\Delta(\bar{\mu})$
T_{00}	$Q-\Delta(\bar{\lambda})$
T_{01}	$\Delta(\bar{\lambda})$
T_{x2}	$\Delta(\bar{\lambda})$ *for* $x=1,\ldots,N-1$
T_{x0}	$diag\{\mu\min(x,w),\ 0\leqslant w\leqslant N\}$, for $1\leqslant x\leqslant N-1$,
T_{x1}	$Q-\Delta(\bar{\lambda})-T_{x0}$ for $1\leqslant x\leqslant N-1$
t_{ij}	the expected transition time from state i to j
M_w	the expected umber of customer in system when w servers working
$\overline{M}$	$[M_0, M_1, \ldots, M_N]$

& de Barros, 2006; Love & Guo, 1996; Monga, Zuo, & Toogood, 1997; Pham & Wang, 1996; Saassouh, Dieulle, & Grall, 2007; Wu & Clements-Croome, 2005), are proposed combined with minimal repairs, unscheduled replacements and other options based on different situations. It can be noted that most maintenance models for single-unit systems mentioned above usually assume that all failures are instantly detected and repaired (Arunraj & Maiti, 2010; Beichelt, 1993; Carazas & Souza, 2010; Kennee et al., 2007; Scarf et al., 2005; Sheu, 1997, 1998; Sheu & Chien, 2004; Sheu & Griffith, 2002). In the real world, this is not always true; what usually happened is that the single-unit system must stop its service and maybe lose the customers waiting in the line. Therefore, this paper will consider the random repair time and multi-unit service systems.

For multi-unit systems, a lot of review papers have been conducted recently (Cho & Parlar, 1991; Dekker, Wildeman, Schouten, & Frank, 1997; Wang, 2002). According to the observation of Wang (2002), maintenance policies for multi-unit systems can be implemented by applying single-unit system maintenance policies for each separate unit respectively if there exists economic independence, failure independence, and structure independence between units within those systems (Aghezzaf & Najid, 2008). The multi-unit systems discussed in this paper sre apparently economic dependent. It means that carrying out maintenance operations on several units simultaneously costs less money or time than on each unit individually. There are various types of maintenance policies for multi-unit systems, such as block replacement policies, opportunistic maintenance policies, or group maintenance policies. For classical block replacement policies, all units in the system are replaced simultaneously at periodic intervals, and each unit failed in between will also be replaced immediately (Barlow & Proschan, 1965; Sun, Xi, Du, & Pan, 2010). Since the classical block maintenance policies are possible to force the system to replace nearly new unfailed units, many studies have developed various modified block maintenance policies to evade this kind of unnecessary waste (Anisimov, 2005; Archibald & Dekker, 1996; Scarf & Cavalcante, 2010; Sun et al., 2010). For opportunistic maintenance policies, upon a unit failure, besides replacing the failed one, other

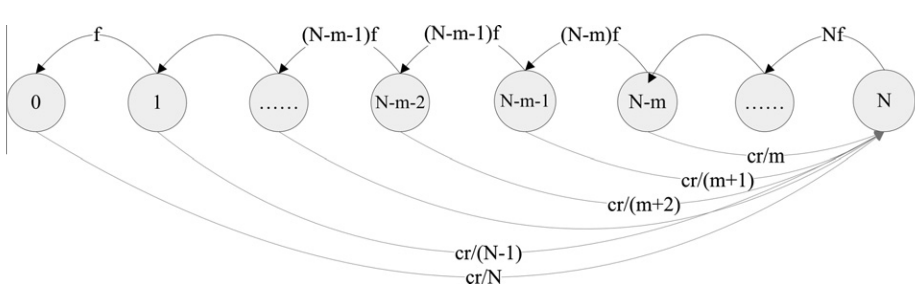

Fig. 1. Transition flow for group maintenance process.