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On-line SPC with consideration of learning curve

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ABSTRACT

This work identifies a link between on-line statistical process control (SPC) and the learning effect for the process standard deviation (PSD) caused by the quality improvement (QI) program. The learning curve (LC) is used to describe and forecast, and the exponentially weighted root mean square control chart is used to monitor the progress in reducing PSD. A modification of the quality control chart (QCC) that considers LC of PSD is proposed. The reduction rate of PSD may be large during the initial stage of the QI program, and influences QCC construction. Simulation is used to compare the shift-detecting ability of the Shewhart-$\bar{X}$ control chart and EWMA-$\bar{X}$ control chart, without- and with- consideration of LC. The EWMA-$\bar{X}$ chart with consideration of LC performs best. In comparison, the Shewhart-$\bar{X}$ chart without LC consideration has almost no shift-detecting ability when the shift magnitude of the process mean is small, leading to rendering quality control ineffective.

1. Introduction

During the mid-1980s, quality improvement (QI) became an important issue for numerous companies. Currently, many organizations are still using the principles and concepts of Total Quality Management, Continuous Improvement, Six-Sigma, etc. to improve quality. Numerous QI programs are based on an administrative structure that implements statistical methods. For example, Six-Sigma is based on the understanding and eliminating causes of variation may simply involve a collection of old well-tried statistical process control (SPC) tools and techniques, but the modern approach has been carefully and professionally adopted by many companies. This approach is now well recognized as an effective method of achieving excellent product and service quality (see Hahn, Doganaksoy, & Hoerl, 2000; Hoerl, 1998).

Some of those QI programs focus on reducing variability in quality characteristics (such as the process standard deviation, PSD) within a specific period. Such as mentioned in Guiffrida and Nagi (2006), the delivery variance in supply chain can be reduced with the context of a continuous improvement program. If an average four sigma (or 6210 defects per million opportunities (DPMO)) company plans to achieve Six-Sigma (or 3.4 DPMO) over a 5 years period (i.e. 60 months) of Six-Sigma deployment, it involves an 11.77% monthly improvement rate. It means that, if d_i denotes the ith month's DPMO after implementing the QI program, $d_i = 6210\ \text{DPMO} \times (1 - 0.1177)^i$, $i = 1, 2, \ldots, 60$, then the 60th month's DPMO, $d_{60} = 6210\ \text{DPMO} \times (1-0.1177)^{60} = 3.4$ DPMO. To reach the target level, the learning curve (LC) can be used to monitor and forecast the rate of reduction of PSD.

LCs have been extensively studied, starting with Wright in 1936, and have been applied in practice (Yelle, 1979). LC provides a method of quantifying, observing and predicting ongoing improvements in manufacturing and service organizations. Numerous scientists and practitioners have observed the strategic importance of LC (Fine, 1986; Jaber, 2006a; Jaber & Bonney, 2003.) The central notion in LC theory is that accumulating experience leads to improved performance, or learning by doing (e.g., Jaber, 2006b). Tapiero (1987) established a linkage between quality control and the learning process involved in production. The author concluded that experience and high quality control move together are correlated through conventional wisdom and more elaboration. Lapre, Mukherjee, and Van Wassenhove (2000) studied the relation between waste rates and quality-based learning. They indicated that waste reduction is a function of the number of QI projects with high conceptual and high operational learning, rather than the total number of improvement projects. Based on detailed data on defect rates and quality cost of the plants, Ittner, Nagar, and Rajan (2001) found that learning is a function of both proactive investments in quality improvement and autonomous learning-by-doing. In this study, we emphasize the learning which comes from quality improvement program, such as Total Quality Management, Continuous Improvement, Six-Sigma, etc. The result of Ittner et al. (2001) is included for supporting this exposition.

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Serel, Dada, Moskowitz, and Plante (2003) noted that investing in reducing process variation is generally more beneficial than reducing the bias between the process mean and the target quality characteristic value. Jaber and Guiffrida (2004) considered learning in production and reworks. They found three behavioral patterns in the quality learning curve: concave, plateau, and monotonically decreasing. Jaber and Guiffrida (2008) investigated the quality learning curve for the assumption that the production process is interrupted for quality maintenance to bring the process in-control again. They showed that the plateauing effect, which could be because of quality problems, can be broken by continuous improvement programs. In this study, we assume that the QI program focuses on the learning strategy that reduces the PSD of the quality characteristic, and helps increase system performance to achieve pre-determined targets.

The quality control chart (QCC), as a major technique of the online SPC, monitors the stability of a process and detects unstable factors. An out-of-control signal indicates the existence of a shift of the mean or variance of the process (caused by the assignable causes) is indicated. The process engineer stops the production line, identifies and eliminates the assignable causes, and restarts production. This process investigation and adjustment is similar with the minor setup mentioned in Khouja (2005). Khouja (2005) reformulated some inventory models which allow for adjustments to restore the process to an "in-control" state. These adjustments were performed without interrupting the system or may require system stoppage and can be thought of as "minor setups". In this study, we assume that the reduction rate of PSD is not affected after the process adjustment.

As mentioned in Montgomery (2005, p. 168), control chart usage generally involves two distinct phases. In phase I, a set of process data is gathered and analyzed for constructing trial control limits and assist operating personnel in bringing the process into a state of statistical control. In phase II, we usually assume that the process is reasonably stable and the major objective of QCC is on process monitoring. Therefore, people generally assume constant PSD, and simply modify the control chart following an extended period until the change in PSD (decrease) is detected based on the gathered data. In the modern high-technology age, some products have extremely short life-cycles (for example only 9 months), and can be manufactured during the early stage of LC. Furthermore, the reduction rate of PSD combined with the QI program is extremely large. Thus without LC consideration, the control limits of QCC are too wide to detect the process shift efficiently during the earlier stage of the QI program.

This study proposes a quality control scheme which establishes a link between on-line SPC and the LC of PSD. To the best of our knowledge, this is the first paper which employs the LC for on-line quality control scheme. The originality and contribution of this paper are doubtless. The quality control scheme considers the LC during the QI program, to maintain appropriate quality control limits and more quickly detect the shift of the process mean. There are various LCs proposed in the literature, such as the exponential LC (e.g., Serel et al., 2003; Zangwill & Kantor, 1998) and the power LC (e.g., Jaber & Bonney, 2003). In this study, an exponential LC model is applied to establish a PSD forecasting model. Any kind of LC can be applied in our proposed quality control scheme. An exponentially weighted root mean square (EWRMS) control chart is then used to monitor the predicted PSD. The improvement information of PSD influences the construction of the QCC.

In the quality control literature (e.g., Saccucci & Lucas, 1990), when designing a control chart for monitoring the process mean, people usually assume the process variance to be constant (in-control). Under the same PSD and pre-determined in-control average run length (ARL_0), with various shifts of the process mean, we can compare the shift-detecting ability of the control charts by out-of-control average run length (ARL_1). If the variation source of PSD also comes from the process shift (out-of-control), the measurement of ARL_1 should be made under both various shifts of the process mean and PSD. It will make the comparison of shift-detecting ability of the control chart more complicate and difficult. In this work, for simplification, we assume that the only variance source of PSD comes from the learning effect. The only variation which contributes to the process is the process mean. Then the sample mean ($\overline{X}$) control charts (such as the Shewhart-$\overline{X}$ control chart and the exponentially weighted moving average (EWMA)-$\overline{X}$ control chart) are adopted for monitoring the process mean of the quality characteristics.

The remainder of this paper is organized as follows. Section 2 uses an exponential learning model to forecast PSD, and uses an EWRMS control chart to monitor PSD. In Section 3 then briefly describes the EWMA-$\overline{X}$ chart. When the smoothing parameter, $\lambda = 1$, the EWMA-$\overline{X}$ chart reduces to the Shewhart-$\overline{X}$ chart. Section 4 presents a simple example for illustration. In Section 5, the performance of the EWMA-$\overline{X}$ chart and the Shewhart-$\overline{X}$ chart, with- and without- LC consideration, and under various shifts of process mean, are compared in terms of average run length (ARL). The relative data are obtained via numerical simulation. Finally, the last section presents conclusions.

2. PSD learning model and the EWRMS control chart

Let X_i denote the value of interested quality characteristic. Suppose that X_i is a variable and comes from a normal $N(\mu_i, \sigma_i^2)$ distribution, where $i = 1, 2, \ldots$, represents the cumulative production quantity following implementing QI program, μ_i is the process mean, and σ_i^2 denotes the process variance (that is σ_i is the value of PSD). To study the PSD, this work collects the information during the setting-up phase (phase I) to yield a preliminary estimate of the asymptotic performance of a manufacturing process (as noted in Franceschini, 2002). The exponential LC in the stable state (phase II) is assumed to be adequate in this study:

$$\sigma_i = \sigma_0 e^{-bi} + \varepsilon, \quad b > 0, \quad i = 1, 2, \ldots \tag{1}$$

where σ_0 denotes the initial level of PDS, b represents the learning rate, and ε is the random error term ($\varepsilon \sim NID(0, \sigma_\varepsilon^2)$). Eq. (1) implies that the PSD, σ_i, decreases at a decreasing rate with increasing i. The Gauss–Newton iteration method (Bates & Watts, 1988) can be used to estimate the parameters in Eq. (1). To predict the process variance at a future cumulative production quantity i following implementing QI program, the following model is used:

$$\hat{\sigma}_i = \sigma_0 e^{-\hat{b}i}. \tag{2}$$

If G^* is the ideal PSD performance after producing T items (that is $\sigma_T = G^* = \sigma_0 e^{-\hat{b}T}$), the learning rate $\hat{b}$ can be obtained as

$$\hat{b} = \frac{1}{T}\ln(\sigma_0/G^*). \tag{3}$$

For instance, if the company plans to decrease σ_i from $\sigma_0 = 2$ (at the beginning of the QI, $i = 0$) to $\sigma_{10,000} = G^* = 1$ (at the end of the QI, $i = T = 10{,}000$), the learning rate will be $\hat{b} = 0.000069314$.

Suppose that samples are obtained at each point in time. Let n denote the size of rational subgroups ($n > 1$), while m represents the items that are produced between subgroups, and $X_i = X_{j,k}$ is the value of the quality characteristic in the jth subgroup, $j = 1, 2, \ldots, k = 1, 2, \ldots, n$. For simplicity, we assume that $X_{j,k} \sim N(\mu_{(j)}, \sigma_{(j)}^2)$. It means that, within the subgroup (say j), $X_{j,k}$ has the same mean (denoted as $\mu_{(j)}$) and the same PSD (denoted as $\sigma_{(j)}$). Moreover, $\sigma_{(j)}$ is set to be the PSD value of the middle observation of the jth subgroup. Hence, according to Eq. (1), $\sigma_{(j)}$ is defined as follows: