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Planar maximal covering with ellipses ☆

Mustafa S. Canbolat *, Michael von Massow

DeGroote School of Business, McMaster University, 1280 Main Street West, Hamilton, Ont., Canada L8S 4M4

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ABSTRACT

We consider a maximal covering location problem on the plane, where the objective is to provide maximal coverage of weighted demand points using a set of ellipses at minimum cost. The problem involves selecting k out of m ellipses. The problem occurs naturally in wireless telecommunications networks as coverage from some transmission towers takes an elliptical shape. A mixed integer nonlinear programming formulation (MINLP) for the problem is presented but MINLP solvers fail to solve it in a reasonable time. We suggest a Simulated Annealing heuristic as an alternate approach for solving the problem. Our computational results employing the heuristic show very good results with efficient processing times. We also discuss an extension to the problem where coverage occurs on a spherical surface and show that it can be solved with the same heuristic.

1. Introduction

Maximal covering location problems (MCLP) come into play when there are insufficient resources to cover all demand points or,alternately, it is also used in cases in which a firm seeks to maximize the return to the coverage selected. Any facility within the coverage distance is covered but outside of the coverage distance is not covered. The problem involves implementing different coverage categories which each have a specific cost associated with them. The objective is to cover the demand points that maximize the return. While the network version of the problem, where facilities are located on the nodes of a network, has been well studied in the literature, its planar counterpart where the facility can be placed anywhere on the plane, has been studied by several authors but is not as extensively explored. This is due to the fact that underlying distance functions make the models complicated.

The problem was first introduced by Church and ReVelle (1974) on a network. They formulate the problem and suggest a solution methodology for a given number of potential facility sites. For a detailed review on the problems studied on the networks, the user is referred to Schilling, Vaidyanathan, and Barkhi (1993) and Daskin (1995). The planar version of the problem was first considered by Mehrez and Stulman (1982) and later by Church (1984). They show that the problem can be solved over a small finite set of points which are the intersection points of circles drawn around demand points and suggested a mixed integer programming (MIP) formulation for the problem. The idea of identifying candidate points can be generalized for all distance norms. For a given distance norm, a corresponding unit ball blown up with a constant factor to the maximum coverage distance can be drawn around demand points to identify possible demand points as intersection points of these shapes. Subsequently, other researchers have considered different coverage distances such as inclined parallelograms (Younies & Wesolowsky, 2004) and block norms (Younies & Wesolowsky, 2007). Others have proposed different coverage functions (Drezner, Wesolowsky, & Drezner, 2004; Karasakal & Karasakal, 2004).

There are also a number of studies that approached to the problem in terms of its practical application. Current and O'Kelly (1992) reported on the application of the problem for locating emergency warning sirens in a case study. The authors considered two siren types with different costs and respective covering radii. Drezner and Wesolowsky (1997) considered locating a number of signal detectors with the objective of maximizing the smallest probability (to maximize the minimum protection) of the detection of an event anywhere on the plane. The authors assumed that the detection probability is a decreasing function of the distance. Recently, Jia et al. introduced a new maximal covering location model for the medical supply distribution for large-scale emergencies (2007a) and suggested heuristics based solution methodologies for the problem (2007b).

In this study we consider another practical usage of the problem. MCLP has great potential when it comes to the problem of

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* Corresponding author. Tel.: +1 905 525 9140x26361; fax: +1 905 521 8995.
E-mail address: canbolat@mcmaster.ca (M.S. Canbolat).

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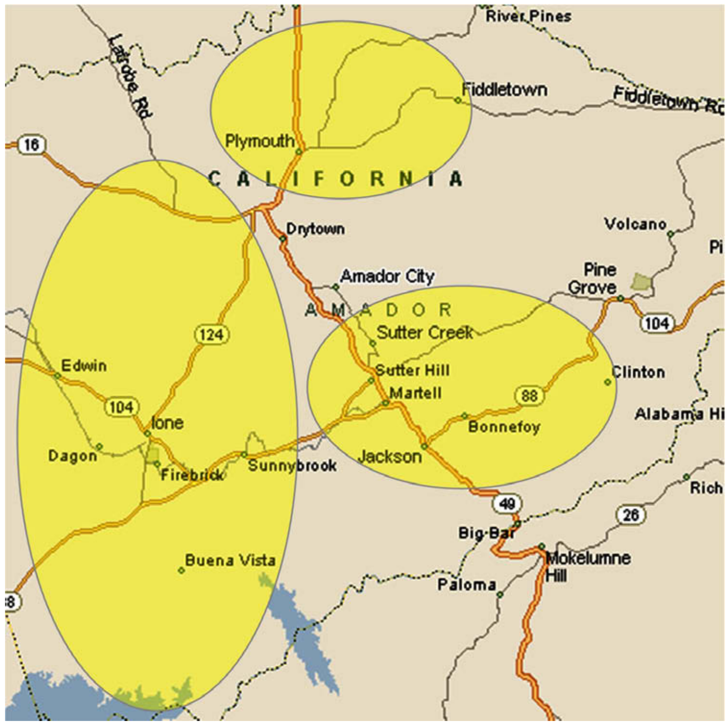

Fig. 1. Wireless high-speed internet coverage areas.

optimizing wireless transmitter coverage. Many satellite and antenna based transmitters have a coverage range that has an elliptical shape. Fig. 1 shows a coverage area for a California based wireless high speed Internet provider.[1]

It is clear from this diagram that companies are making decisions as described above and excluding some areas from coverage. Aguado-Agelet, Varela, Alvarez-Vazquez, Hernando, and Formella (2002) suggest that optimization techniques have not been widely used in transmitter location decisions. They introduce some solution approaches in a model that coverage requirements are predetermined and the objective is to locate transmitters such that power transmission is minimized but all of the required area is covered. It does not appear that selecting the coverage area based on minimum cost has been considered in the communications literature. To the best of our knowledge, despite its practical importance, MCLP by ellipses has not been considered in facility location literature. While most of the papers that consider circular (Euclidean) coverage, discuss antenna transmissions as an application of the problem, they do not acknowledge the fact that the circular maximal coverage is an approximation to the elliptical coverage. Our approach, therefore, provides a more precise and accurate solution than has been used before for the wireless transmission location problem. It also provides the flexibility to have ellipses with different parameters depending of the specific technical characteristics of the transmitter.

In this paper we provide a mixed integer nonlinear programming (MINLP) formulation for multi-facility MCLP with ellipses (MCLPE). We find that available solvers fail to provide a solution within a reasonable time. We then suggest a Simulated Annealing (SA) heuristic for the problem and show that it performs efficiently making the improved, more accurate approach feasible.

The paper is organized as follows. In Section 2 we present the MINLP formulation for the problem. This section includes a discussion of the geometric properties of ellipses in order to set the context for the formulation. Section 2.3 contains an example problem with computational results. In Section 3 we present the SA heuristics for the problem. We provide computational results to show that the approach performs well. In Section 4 we discuss the extension in which we evaluate spherical MCLP by ellipses. Finally in Section 5 we conclude and provide future research directions.

2. Problem definition and formulation

2.1. Problem definition

The problem involves n demand points, each with a nonnegative weight $w_j, j = 1, \cdots, n$, present in the plane. In the context of the transmitter location problem, a demand point could represent a population center and the weight the number of potential customers. We are trying to locate k elliptically shaped facilities from among a set of m facilities with a fixed elliptical coverage distance with parameters $((a_i, b_i), i = 1, \cdots, m)$. That is, we are choosing specific transmitters from among a possible technology set. The parameters (a_i, b_i) represent the semi-major and semi-minor axes of the facility i respectively, at locations defined by their foci (f_{1i}, f_{2i}) such that the facilities cover the maximum number of demand points (maximum weight) at these points. The parameters (a_i, b_i) determine the size and specific shape of the ellipse and are given. The foci (f_{1i}, f_{2i}) are the decision variables and set the location. Each facility has a cost of selection c_i.

The formulation of the problem makes use of the geometric properties of an ellipse. From these geometric properties outlined below, we derive the conditions of the coverage.

2.1.1. The geometric properties of an ellipse

The shape of an ellipse is expressed by a number called the eccentricity of the ellipse, ε. The eccentricity is a positive number

[1] http://communicationsadvantage.net/internet/coverage.html