Computers & Industrial Engineering 57 (2009) 401–407

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Monitoring process mean and variability with generally weighted moving average control charts ☆

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ARTICLE INFO

Article history:
Received 9 April 2007
Received in revised form 27 September 2008
Accepted 28 December 2008
Available online 31 January 2009

Keywords:
Exponentially weighted moving average
Simulation
Average run length
Generally weighted moving average
Control chart

ABSTRACT

This study investigates control charts for simultaneous monitoring of process mean and process variability when an individual observation is taken at each sampling point. A combined scheme consisting of a two-side generally weighted moving average (GWMA) mean chart and a two-side GWMA variance chart is developed. This new combined scheme will compare with the exponentially weighted moving average (EWMA) single charts and a combined EWMA chart. It is shown that the combination of the GWMA charts is more sensitive than the combination of the EWMA charts for detecting small shifts in the process mean and variance. An example and a simple procedure for the design of a combined GWMA chart are also given to illustrate this study.

1. Introduction

The modern statistical process control (SPC) can be said to have been born when Walter A. Shewhart, a physicist and statistician working for Bell Laboratories, developed the concept of a control chart in the 1920s. Quality engineers monitor quality characteristics (product attributes) using control charts to improve processes. In SPC, a production process is thought of being executed in either of two mutually exclusive states: an in-control state, and an out-of-control state. Shewhart developed graphical devices called control charts to distinguish between these two states. As long as the chart does not indicate the existence of an out-of-control state, the process is considered to be operating in statistical control. An important objective of statistical process monitoring and control is the fast detection of variation in the production system to enable necessary corrective actions to be taken before even more defective items are produced.

Advanced process monitoring techniques, such as exponentially weighted moving average (EWMA) or cumulative sum (CUSUM) control charts can be used in automated operating environments where appropriate. Roberts (1959) showed that the EWMA control chart is useful for detecting small shift in the mean of a process. Control charts for monitoring the process mean and the process variability are often based on samples of $n > 1$ observations, but in many applications individual observations are used (Reynolds & Stoumbos, 2004). In their overall conclusion is that it is best to take sample of $n = 1$ observations and use an EWMA or CUSUM chart combination. The use of the EWMA was applied to individual observations by Wortham and Heinrich (1972), who point out that such an approach may be justified when the cost of inspection is high or when expensive destructive testing is involved. Montgomery (2001) pointed out that in computer integrated manufacturing where sensors are used to measure every unit manufactured, the subgroup size was set to $n = 1$.

Control charts are basic and powerful tools in statistical process control and are widely used to control various industrial processes. It is very important that control charts can quickly detect the out-of-control signals when the process mean shifts. The method of Sheu and Griffith (1996) and Sheu (1998), Sheu (1999) is applied to EWMA control charts to enhance the detection ability of control charts. The expanded chart of Sheu and Lin (2003) is called the generally weighted moving average (GWMA) control chart. Their simulation results indicated that GWMA is more sensitive than EWMA in detecting small shifts in the process mean. Sweet (1986) recommended using two EWMA charts, one to detect mean shifts and the other to detect changes in variance. Reynolds and Stoumbos (2001) considered the use of two EWMA charts to make individual observations. This study extends the GWMA control chart that meets some of the above requirements.

☆ This manuscript was processed by Area Editor E.A. Elsayed.
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doi:10.1016/j.cie.2008.12.010

2. Mean charts and variance charts

The control chart technique has been widely applied in manufacturing industries because its chart is easy to plot, easy to interpret, and its control limits are easy to obtain (DeVor, Chang, & Sutherland, 1992; Duncan, 1986; Gitlow, 1995; Grant & Leavenworth, 1988; Mitra, 1993; Montgomery, 2001).

Reynolds and Stoumbos (2001) were shown that the combination of the EWMA charts is more sensitive than the combination of the X and MR charts for detecting small shifts in the process mean and variance. Thus, we are primarily interested in comparing the GWMA charts with the EWMA charts. These mean charts and variance charts are defined as follows. Here, X_t is assumed to be independent and identically distributed that was obtained from the t-th sample. In fact, X_t denotes the value of the quality characteristic, the parameter μ is the process mean, and the parameter σ^2 is the process variance. When the process is in-control, let μ_0 represent the value for μ and σ_0 represent the value for σ.

2.1. EWMA mean chart

The EWMA mean chart for detecting shifts in μ is based on the statistic

$$Z_t = (1-\lambda)Z_{t-1} + \lambda X_t, \quad t = 1, 2, \ldots$$

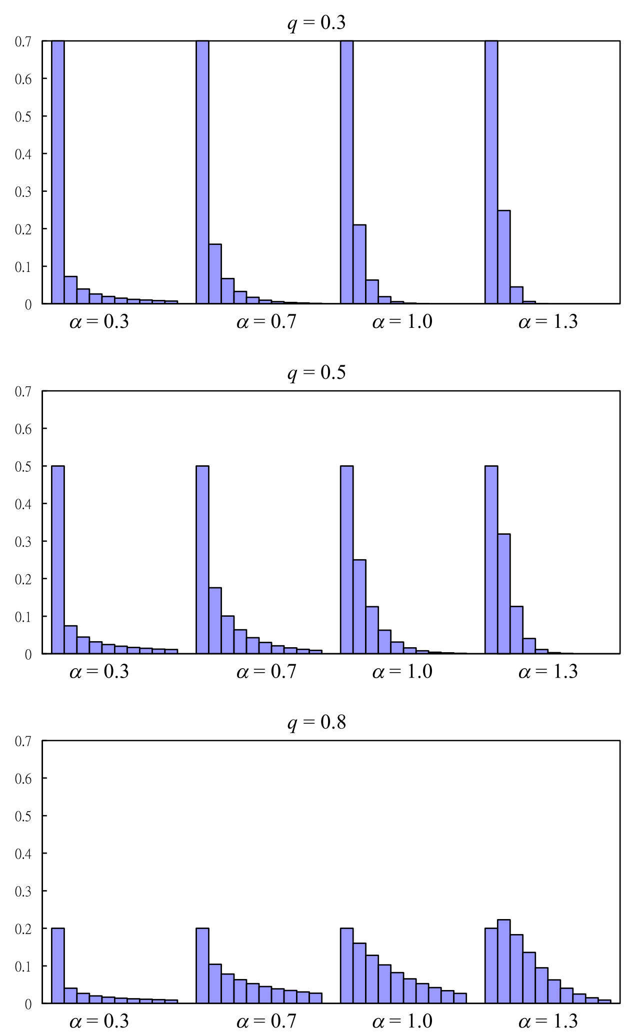

Fig. 1. The example of weighting function for a GWMA.

where λ is a weight parameter such that $0 < \lambda \leqslant 1$,and the starting value is usually $Z_0 = \mu_0$. where $\lambda = 1$, the EWMA chart reduces to the Shewhart X chart. To monitor the process, the averages Z_t are plotted in time order on a EWMA chart with the upper and lower control limits as

$$\mu_0 \pm L_Z \sigma_0 \sqrt{\lambda/(2-\lambda)}$$

where $\sigma_0\sqrt{\lambda/(2-\lambda)}$ is the asymptotic standard deviation of Z_t, and a signal is given if Z_t falls outside of control limits. This EWMA mean chart based on Z_t will be represented as the EWMA$_Z$ chart.

2.2. GWMA mean chart

A statistic of the GWMA technique for detecting shifts in μ is defined as

$$Y_t = \sum_{i=1}^{t} \left(q^{(i-1)^\alpha} - q^{i^\alpha}\right) X_{t-i+1} + q^{t^\alpha}\mu_0, \quad t = 1, 2, \ldots$$

where $0 \leqslant q \leqslant 1$, $\alpha > 0$, and the starting value is usually $Y_0 = \mu_0$ (Sheu & Lin, 2003). The design parameter q is constant, and we can use the parameter α to adjust the kurtosis of weighting function slightly. Fig. 1 displays the example of weighting function for a GWMA.

When $q = 1 - \lambda$, the weight that is given to the current sample in GWMA is the same as EWMA. When $q = 1 - \lambda$ and $\alpha = 1$, the weighting function of GWMA is the same as EWMA, and the GWMA chart reduces to the EWMA chart. When $q = 0.0$, $\alpha = 1$ and the width of the control limits $L_Y = 3.0$, then the GWMA chart reduces to the Shewhart X chart. Fig. 2 displays the relationships among the Shewhart X chart, the EWMA chart and the GWMA control chart.

Using large q and $0.5 \leqslant \alpha \leqslant 1$, the GWMA mean chart will be more effective than the EWMA mean chart in detecting small shifts in μ (Sheu & Lin, 2003). Therefore, the control limits for the GWMA control chart are as

$$\mu_0 \pm L_Y \sigma_0 \sqrt{W_L}$$

where $W_L = \lim_{t\to\infty}\left\{\sum_{i=1}^{t}\left(q^{(i-1)^\alpha} - q^{i^\alpha}\right)^2\right\}$, and $\sigma_0\sqrt{W_L}$ is the asymptotic standard deviation of Y_t. The process is considered out of control and some action should be taken whenever Y_t falls outside the range of the control limits. This GWMA mean chart based on Y_t will be represented as the GWMA$_Y$ chart.

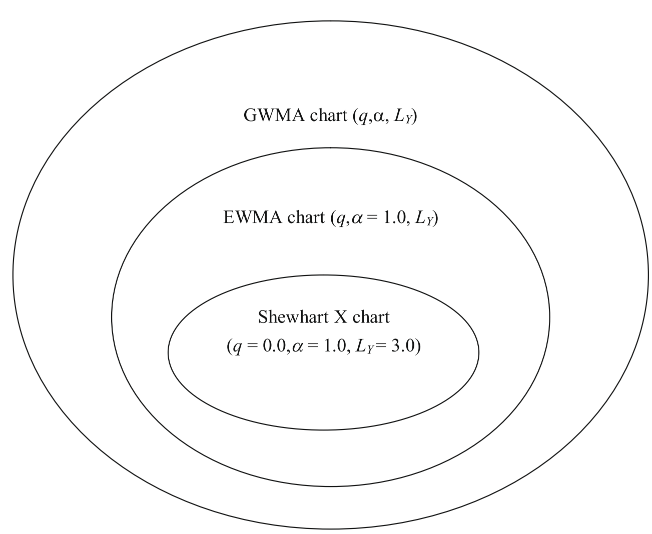

Fig. 2. GWMA, EWMA and Shewhart X control chart.