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Incorporating uncertainty in optimal decision making: Integrating mixed integer programming and simulation to solve combinatorial problems

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ABSTRACT

We introduce a novel methodology that integrates optimization and simulation techniques to obtain estimated global optimal solutions to combinatorial problems with uncertainty such as those of facility location, facility layout, and scheduling. We develop a *generalized* mixed integer programming (MIP) formulation that allows iterative interaction with a simulation model by taking into account the impact of uncertainty on the objective function value of previous solutions. Our approach is generalized, efficient, incorporates the impact of uncertainty of system parameters on performance and can easily be incorporated into a variety of applications. For illustration, we apply this new solution methodology to the NP-hard multi-period multi-product facility location problem (MPP-FLP). Our results show that, for this problem, our iterative procedure yields up to 9.4% improvement in facility location-related costs over deterministic optimization and that these cost savings increase as the variability in demand and supply uncertainty are increased.

1. Introduction

The negative effect of various sources of uncertainty on operational performance (e.g., cost, profitability, quality, and customer service) is well documented in the literature (Acar, 2007; Hahn, Duplaga, & Hartley, 2000; Lee & Billington, 1995; Lee, Padmanabhan, & Whang, 1997; Qi, Bard, & Yu, 2004; Vidal & Goetschackx, 2000). Yet, inclusion of uncertainties (e.g., demand, lead time, production yield, and raw materials cost) often makes pure mathematical modeling intractable. Therefore, in practice, deterministic mathematical models are widely employed and are followed by multiple sensitivity analyses to evaluate the impact of various types of uncertainty on operational performance.

Another widely used business decision tool is simulation where uncertainty in various system parameters can be incorporated; however, it is useful for evaluating the operational performance of only a particular scenario. Simulating all possible scenarios often requires too much computation time for a business problem of realistic size, even with today's computer power. For example, in a facility location problem (FLP), the total number of possible configurations to be simulated is $2^n - 1$, where n is the number of facilities. Considering the advantages and disadvantages of these two techniques, some researchers have adopted hybrid approaches incorporating both mathematical optimization and simulation techniques (Acar, 2007; Butler, Karwan, & Sweigart, 1992; Dijk & Sluis, 2006; Lee et al., 2006; Lin et al., 2000; Moore, Warmke, & Gorban, 1991; Muriel, Anand, & Yongmei, 2006; Qi & Bard, 2006; Smith et al., 2007; Tang & Liu, 2007). A few researchers have used these two techniques iteratively, by returning values from simulation for re-optimization to solve specific problems, exchanging problem-specific parameters between the two techniques (Byrne & Hossain, 2005; Carlson, Hershey, & Kropp, 1979; De Angelis, Felici, & Impelluso, 2003; Karabakal, Gunal, & Ritchie, 2000; Ko, Ko, & Kim, 2006; Lee, Kim, & Moon, 2002; Leung & Cheung, 2000; Leung, Maheshwari, & Miller, 1993; Nolan & Sovereign, 1972).

This paper introduces a novel solution methodology that integrates optimization and simulation and is easily adaptable to various combinatorial problems such as those of facility location, facility layout, and scheduling. Our solution methodology differs from those set forth in previous research in that we present a *general* solution methodology that can obtain a global optimum for business problems involving various sources of uncertainty. Our solution methodology incorporates a *generalized* MIP to obtain estimated optimal solutions to the business problem of interest.

The paper is organized as follows: In Section 2, we provide a literature review of hybrid methods that iteratively integrate optimization and simulation methodologies. In Section 3, we describe our new hybrid approach. In Section 4, we present the illustration of

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our solution methodology on the multi-period multi-product facility location problem (MPP-FLP). Finally, in Section 5, we present our conclusions.

2. Literature review

Several researchers have developed iterative solution approaches for various types of problems that integrate optimization and simulation methodologies. To date, these solution methods have focused on solving very specific problems when incorporating uncertainty factors.

The earliest solution approach involving iterative use of simulation and optimization was developed by Nolan and Sovereign (1972). Their recursive approach involves an allocation of resources by a linear programming (LP) model at an aggregate level and a revision of productivity estimates by simulation of the schedules generated by optimization. They applied their solution approach to the strategic mobility system problem of the US military transportation system.

Leung et al. (1993) developed an iterative approach for flexible manufacturing systems planning. The inputs of their integer programming optimization model include system utilization, makespan, and vehicle utilization. These parameters are updated by simulating the output of the optimization before resolving again in an iterative fashion. Their procedure terminates when the simulation outcomes comply with the results from the optimization.

Leung and Cheung (2000) developed a hybrid iterative solution methodology for the DHL Hong Kong distribution network. Their simulation model is used to evaluate the daily operational performance of the network configuration suggested by their MIP optimization. If service coverage or service reliability is unacceptable, or if the utilization and cost estimates differ significantly from those used in the optimization model, the input parameters are updated and the optimization model is solved again.

Karabakal et al. (2000) developed a hybrid solution approach that iterates between a simulation and MIP model with an objective to find the optimum configuration for the Volkswagen of America's vehicle distribution system. Two major input parameters to the MIP are demand and truck load factors, both of which depend on the location policy. Therefore, simulation is used to generate demand and truck load factor estimates as a result of implementing a particular location policy obtained by the MIP model. They used these estimates to update the input parameters (demand and truck load factor) of the MIP in an iterative manner until both the MIP and simulation agreed on a particular location policy.

Lee et al. (2002) developed a hybrid solution approach that combines analytic LP and simulation models to solve multi-product and multi-period production–distribution problems. Their LP model minimizes the overall cost of production, distribution, inventory holding, and shortage costs and determines production and shipment quantities between production facilities and retailers. Their simulation model is used to adjust the production time and distribution lead-time in the LP model. The iteration stops when the difference between the preceding and current simulation runs in the production and distribution lead-times is deemed small enough.

De Angelis et al. (2003) developed a solution methodology that interactively uses simulation and optimization to determine the estimated optimal configuration of servers in a health care facility. In their iterative solution approach, simulation is used to generate a "training set" from which a relationship between the input parameters (number of servers at each service location) and resulting service performance (average time spent in system) is estimated and then used as an objective function in the optimization. They adopted a radial basis function, a particular type of neural network, to estimate this relationship. The configuration obtained by solving the optimization model is simulated. If the difference in the solution value obtained by simulation and optimization models is small enough, the procedure terminates. Otherwise, the configuration is added to the training set and a new estimation of the objective function is calculated to be used in the next optimization model.

Byrne and Hossain (2005) developed an extended linear programming model for the hybrid modeling approach first proposed by Byrne and Bakir (1999). Their hybrid solution approach iteratively applies simulation and LP to solve a multi-period multi-product production planning problem. They obtained job workload and resource utilization through simulation and used LP to obtain the estimated optimal production plan that minimizes total costs. Byrne and Bakir (1999) demonstrated that their solution approach outperforms an LP approach alone.

Ko et al. (2006) developed a hybrid optimization-simulation approach to design a distribution network for third party logistics (3PL) providers. They used a genetic algorithm to solve the optimization model that determines the distribution network structure. Subsequently, the simulation model is applied to capture the uncertainty in client demand, order-picking time, and travel time for the capacity plans of the warehouses. The simulation is used to estimate the average service time at each warehouse. Then, the service time is used to define appropriate throughput capacity constraints to be incorporated into subsequent optimization runs. If the simulation outputs satisfy the required performances, the procedure is terminated.

Each of the iterative solution procedures described above is specific to a particular problem and exchange problem-specific parameters between simulation and optimization models. We present and examine a general solution methodology that obtains an estimated global optimum for combinatorial optimization problems with components of uncertainty.

3. Hybrid solution methodology

We propose a novel solution methodology that can be employed to solve various combinatorial problems with components of uncertainty. A general MIP formulation is developed to obtain the estimated optimal solution to the deterministic problem. Then, a simulation of the deterministic solution determines the impact of uncertainty on the objective. The difference between the deterministic and simulated objective function is incorporated back into the MIP formulation. As a new solution is searched and alternatives are evaluated by the MIP model, solutions that were previously obtained and simulated are evaluated by taking into account the impact of uncertainty on the objective. The process iterates until a previously simulated solution with uncertainty impact is found to be optimal in the current MIP formulation.

The steps of the solution procedure are summarized in Fig. 1. For simplicity, we assume that the objective function is a cost minimization.

3.1. Step 1: Run MIP optimization

The following generalized MIP formulation incorporates the necessary additional components for integrating the resulting uncertainty impact into the optimization procedure. The formulation below assumes a cost minimization, but a similar formulation is easily created for profit maximization or any other objective. We assume that for any potential solution, the deterministic objective will be less than the objective obtained under uncertainty. Although it is uncommon that uncertainty will, in fact, lower cost, this result is possible. Nonetheless, a deterministic objective less than that found under uncertainty can be ensured by using prob-