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Optimal production run length in deteriorating production processes with fuzzy elapsed time

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ABSTRACT

This paper investigates the optimal production run length in deteriorating production processes, where the elapsed time until the production process shifts is characterized as a fuzzy variable, also the setup cost and the holding cost are characterized as fuzzy variables, respectively. A mathematical formula representing the expected average cost per unit time is derived, and some properties are obtained to establish an efficient solution procedure. Since there is no closed-form expression for the optimal production run length, an approximate solving approach is presented. Finally, two numerical examples are given to illustrate the procedure of searching the optimal solutions.

1. Introduction

The economic production quantity (EPQ) model has been used for a long time and is widely accepted and implemented in practice because of its simplicity (Bedworth & Bailey, 1987). In the basic EPQ model, one of the assumptions is that all the items produced are of perfect quality. However, in real production activities, failure-free production facilities are rare, and the product quality is not always perfect and usually depends on the state of the production process. In most manufacturing industries, production systems usually deteriorate continuously due to usage or age factors such as corrosion, fatigue and cumulative wear. In the simplest characterization, the process state is classified as either "in-control" or "out-of-control". Generally, a production process starts to produce in an in-control state; after a period of time, owing to the continuous deterioration, the process may shift from this "in-control" state to an "out-of-control" state in which some nonconforming items are produced, and these nonconforming items must be rejected, repaired or reworked. Thus substantial costs will be incurred. In the literature, Rosenblatt and Lee (1986) considered the optimal production run length and the effect of the system deterioration on the expected cost rate when the time to shift is exponentially distributed and the maintenance cost is negligible. They showed that the optimal production run length is shorter than that in the classical EPQ model because smaller lot produces fewer defective items. Porteus (1986) described a system that begins each production run in control. As each unit is produced, there is a probability p that the system goes out of control, at which time all subsequent units are defective. The time until the process going out of control therefore follows a geometric distribution. Porteus (1986) also found the similar results to the Rosenblatt and Lee model. Kim and Hong (1999) generalized the Rosenblatt and Lee model by assuming that an elapsed time until shift is arbitrarily distributed, and derived the optimal production run length and minimum average cost in three deteriorating processes: constant, linearly increasing, and exponentially increasing, respectively. Hariga and Ben-Daya (1998) considered general time to shift distributions and provided distribution-based and distribution-free bounds on the optimal cost. For the exponential case, they compared the optimal solutions to approximate solutions proposed in the literature and conducted a sensitivity analysis to see the effect of the input parameters on the various solutions to the problem. Freimer, Thomas, and Tyworth (2006) investigated the effect of imperfect yield on economic production quantity decisions, where the production system is assumed to produce some time-varying proportion of defective parts which can be repaired at some unit cost. For a general defect rate function, they also developed results that characterize the optimal run length and expected total cost and how these objects are affected by the cost parameters.

The imperfect production processes have also been studied with considerations of inspection, maintenance, warranty and investment in setup cost reduction and in process quality improvement. For example, the problem of joint determination of optimal

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production run length and inspection schedule was considered by Kim and Hong (2001), Lee and Rosenblatt (1985, 1987) and Wang and Sheu (2001). The problem of joint determination of optimal production run length and warranty policy was studied by Yeh, Ho, and Tseng (2000), Yeh and Chen (2006). Recently, Hou (2007) considered an economic production quantity model with imperfect production processes, in which the setup cost and process quality are functions of capital expenditure. The optimal capital investment strategies in setup reduction and process quality improvement were presented, but the optimal run length obtained in Hou (2007) is not true for the authors could not prove the monotonic of $f(t)$ in $t \in (0, +\infty)$. In fact, as noted in Freimer et al. (2006), the investment in setup cost reduction results in a reduction in the number of defects produced, but the total number of defects can increase or decrease with an investment in quality improvement.

In the literature mentioned above, the shift time from in-control state to out-of-control state was assumed to be a random variable and the expected average cost was evaluated by using the probability measure. Although this assumption has been adopted and accorded with the facts in widespread cases, it is not reasonable in a vast range of situations. In some production processes, the estimations of probability distributions of system and components are very difficult due to uncertainties and imprecisions of data. instead, fuzzy theory can be employed to handle these cases. Moreover, we note that in the researches mentioned above, the cost parameters such as the setup cost, holding cost were fixed constants. However, in real applications, these cost parameters are hard to express accurately for the lack of historical data, and it is reasonable to characterize them as linguistic or fuzzy variables depending on managers' judgments or experience. In this paper, we introduce the credibility theory into the study of the imperfect production process and explore the problem in the fuzzy sense. The rest of this paper is organized as follows. In Section 2, some preliminaries are introduced. In Section 3, the optimal production run lengths in two cases are investigated, and some theoretical results are presented. In order to solve the optimal run length, an approximate solving approach is presented. Two numerical examples are given in Section 4 to illustrate the procedure of searching the optimal solutions.

2. Fuzzy variable

Since the pioneer work of Zadeh (1965), fuzzy theory has been developed rapidly, and some new concepts and ideas are teemed. After defined the concepts of possibility measure and necessity measure, Zadeh also established the possibility theory (Zadeh, 1978, 1979). These two measures have been explored and used widely (Dubois & Prade, 1988). However, neither possibility measure nor necessity measure has self-duality property. Whereas, a self-dual measure like probability is absolutely needed in both theory and practice. Recently, Liu and Liu (2002) presented the concept of credibility measure, which has self-duality property. Moreover, as a new branch of mathematics, credibility theory was initiated by Liu (2004) to study the behavior of fuzzy events. A detailed survey on credibility theory may be found in Liu (2006). In this section, we will recall some concepts as follows.

Let Θ be a nonempty set, $\mathscr{P}(\Theta)$ the power set of Θ. For an element $\mathscr{A}$ in $\mathscr{P}(\Theta)$, $\mathrm{Cr}\{\mathscr{A}\}$ expresses the chance that fuzzy event $\mathscr{A}$ occurs and is called a credibility measure (see Liu, 2004). In addition, the triplet $(\Theta, \mathscr{P}(\Theta), \mathrm{Cr})$ is called a credibility space, and a fuzzy variable is defined as a function from the credibility space $(\Theta, \mathscr{P}(\Theta), \mathrm{Cr})$ to the set of real numbers (see Liu, 2004, 2006). Let ξ be a fuzzy variable defined on $(\Theta, \mathscr{P}(\Theta), \mathrm{Cr})$. Then its membership function is derived from the credibility measure by

$$\mu(x) = (2\mathrm{Cr}\{\xi = x\}) \wedge 1, \quad x \in \Re. \tag{1}$$

Definition 1 (Liu & Liu, 2002). A fuzzy variable ξ is said to be

(a) nonnegative if $\mathrm{Cr}\{\xi < 0\} = 0$;
(b) positive if $\mathrm{Cr}\{\xi \leqslant 0\} = 0$;
(c) continuous if $\mathrm{Cr}\{\xi = x\}$ is a continuous function of x.

Definition 2 (Liu & Liu, 2002). Let ξ be a fuzzy variable on the credibility space $(\Theta, \mathscr{P}(\Theta), \mathrm{Cr})$. Then the expected value $E[\xi]$ is defined as

$$E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \geqslant r\}\mathrm{d}r - \int_{-\infty}^{0} \mathrm{Cr}\{\xi \leqslant r\}\mathrm{d}r, \tag{2}$$

provided that at least one of the two integrals is finite. Especially, if ξ is a positive fuzzy variable, then $E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \geqslant r\}\mathrm{d}r$.

Definition 3 (Liu & Gao, 2007). The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_m$ are said to be independent if

$$\mathrm{Cr}\left\{\bigcap_{i=1}^{m}\{\xi_i \in B_i\}\right\} = \min_{1 \leqslant i \leqslant m} \mathrm{Cr}\{\xi_i \in B_i\}, \tag{3}$$

for any sets $B_1, B_2, \ldots, B_m$ of $\Re$.

Proposition 1 (Liu & Liu, 2003).*Let ξ and η be independent fuzzy variables with finite expected values. Then for any numbers a and b, we have*

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]. \tag{4}$$

Definition 4 (Liu, 2002). Let ξ be a fuzzy variable on the credibility space $(\Theta, \mathscr{P}(\Theta), \mathrm{Cr})$. Then the credibility distribution $\Phi : \Re \to [0, 1]$ of the fuzzy variable ξ is defined by

$$\Phi(x) = \mathrm{Cr}\{\theta \in \Theta | \xi(\theta) \leqslant x\}, \quad x \in \Re. \tag{5}$$

That is, $\Phi(x)$ is the credibility that the fuzzy variable ξ takes a value less than or equal to x.

Definition 5 (Liu, 2002). Let ξ be a fuzzy variable on a credibility space $(\Theta, \mathscr{P}(\Theta), \mathrm{Cr})$. Then the credibility density function $\phi : \Re \to [(0, +\infty)$ of the fuzzy variable ξ is a function such that

$$\Phi(x) = \int_{-\infty}^{x} \phi(y)\mathrm{d}y, \quad \forall x \in \Re, \int_{-\infty}^{+\infty} \phi(y)\mathrm{d}y = 1, \tag{6}$$

where Φ is the credibility distribution of the fuzzy variable ξ.

Example 1. For a triangular fuzzy variable (a,b,c), whose membership function is given by

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \leqslant x \leqslant b \\ \frac{x-c}{b-c}, & \text{if } b < x \leqslant c \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

From Definition 4, its credibility distribution is

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leqslant a \\ \frac{x-a}{2(b-a)}, & \text{if } a < x \leqslant b \\ \frac{x-2b+c}{2(c-b)}, & \text{if } b < x \leqslant c \\ 1, & \text{if } c \leqslant x. \end{cases} \tag{8}$$