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Mis-specification analysis between normal and extreme value distributions for a screening experiment

Hong-Fwu Yu *

National Kaohsiung University of Applied Sciences, Graduate Institute of Commerce, 415 Chien Kung Road, Kaohsiung 807, Taiwan, ROC

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A B S T R A C T

Design of experiments (DOEs) are useful techniques for improving the reliability (or quality) of a product. The main work of a DOE is to select significant factors that affect the product reliability (or quality). Then the significant factors can be set at the levels which lead to reliability improvement. One of the basic assumptions of DOEs is that the (logged) observations at each run follow a normal distribution. In practical applications, normal and extreme value distributions are much alike. They may fit the data at hand well in practical applications. However, their predictions may lead to a significant difference. A well-known assertion: "moderate departures from normality are of little concern in the fixed effects analysis of variance" [Montgomery, D. C. (1997). Design & analysis of expremients (4th ed.). New York: Wiley]. The main purpose of the present paper is to evaluate the assertion by investigating the impact of mis-specification between normal and extreme value distributions on the precision of selecting significant factors for a screening experiment. For each of these two distributions, the probabilities of correct and incorrect selections under correct specification and mis-specification are computed. The results indicate that for both of normal and extreme value distributions, the selection precision is significantly influenced by mis-specification. An example is used to illustrate the proposed method. Finally, some numerical results are provided to evaluate the impacts of mis-specification on the selection precision for the screening experiment. Th numerical results indicate that for both of normal and extreme value distributions, the smaller the main effect and the sample size, the more the impact of mis-specification is. Surprisingly, this seems to violate the assertion stated above.

1. Introduction

Owing to the strong competition in markets, continuously improving the reliability (or quality) of a product has become a necessary condition for the manufacturer to compete with others. Design of experiments (DOEs) are useful techniques for improving the reliability (or quality) of a product. The main work of a DOE is to select significant factors that affect the product reliability (or quality). Then the significant factors can be set at the levels which lead to reliability improvement. One of the basic assumptions of DOEs is that the (logged) observations at each run follow a normal distribution. In practical applications, normal and extreme value distributions are much alike. They may fit the data at hand well in practical applications. However, their predictions may lead to a significant difference.

As stated above, the analyst is usually confronted with the problem of selecting an appropriate distribution based on whatever data are available. Many excellent researchers have given considerable attention to such a problem of discriminating between two distributions for some given observations. For example, White (1982) investigated the problem of model mis-specification when maximum likelihood techniques are used for estimation and inference. He also obtained the asymptotic properties of the quasi-maximum likelihood estimator with mis-specification under some regular conditions. Atkinson (1969, 1970), Chen (1980), Chambers and Cox (1967), Cox (1961, 1962), Jackson (1968), and Dyer (1973) studied the discrimination problem in general between two models. Besides, Jackson (1969), Quesenberry and Kent (1982), and Wiens (1999) studied the discrimination problem between lognormal and gamma distributions. Bain and Englehardt (1980) and Fearn and Nebenzahl (1991) studied the discrimination problem between Weibull and gamma distributions. Gupta and Kundu (2003) considered the discrimination problem between Weibull and generalized exponential distributions. Gupta and Kundu (2004) considered the discrimination problem between gamma and generalized exponential distributions. Kundu, Gupta, and Manglick (2005) considered the discrimination problem between lognormal and generalized exponential distributions.

Among the discrimination problems, the one for Weibull and lognormal distributions is particularly important and has received much attention, this is because the two distributions are the most

* Tel.: +886 7 3814526x6706.
E-mail address: yuhf@cc.kuas.edu.tw

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popular ones for analyzing the lifetime of electronic products. Dumonceaux and Antle (1973) adopted the ratio of maximized likelihood (RML) in discriminating between the two distributions for complete data, and provided the percentile points for some sample sizes by simulation. Recently, Kundu and Manglick (2004) considered the discrimination problem for complete data using the RML procedure. Based on the result of White (1982), they obtained the asymptotic distribution of the logarithm of the RML and determined the sample size required to discriminate between the two distributions for a pre-specified probability of correct selection. Although the results given in the aforementioned papers are interesting and valuable, it is a pity that the impacts of mis-specification between Weibull and lognormal distributions on the estimation or inference in other statistical models (e.g., design of experiment) are not studied. The main purpose of the present paper is to address this issue. More specifically, for a screening experiment, the essence of this study is to investigate the impacts of mis-specification between lognormal and Weibull distributions on the precision of selecting significant factors. For each of these two distributions, the probabilities of correct and incorrect selections under correct specification and mis-specification are computed. Because discrimination between lognormal and Weibull distributions is equivalent to that between normal and extreme value distributions, we will focus our attention on the latter in the following sections.

The rest of this paper is organized as follows. Section 2 introduces a motivating example. Section 3 briefly describes the assumptions of a screening experiment and the measure of selection quality. Section 4 presents the estimation of parameters for normal and extreme value distributions. Section 5 evaluates the probabilities of correct and incorrect selections with correct specification. Section 6 evaluates the probabilities of correct and incorrect selections with mis-specification. Section 7 applies the proposed method to the motivating example introduced in Section 2 to illustrate the impact of mis-specification between normal and extreme value distributions. Section 8 investigates numerically the impact of mis-specification on the selection precision for the screening experiment described in Section 3. Finally, we make a conclusion in Section 9.

2. Motivating example

To improve the reliability of fluorescent lamps, Tseng, Hamada, and Chiao (1995) conducted a $2^{(3-1)}$ fractional factorial experiment to study three two-level factors (denoted by $-$ and $+$ for low and high, respectively) in four different combinations of factor levels. Because of limited testing equipment availability, only five lamps from a production run of each of the four lamp types were randomly selected for testing.

The luminous flux (in lumens) for each of the 20 lamps were measured at 100, 500, 1000, 2000, 3000, 4000, 5000, and 6000 h, and only the readings for lamps from Runs 2 and 4 were continued up to 12,000 h at 1000 h intervals. Note that the first 100 h is a burn-in time and that the reading at 500 h is useful for detecting defective lamps. The following log-linear model was used to describe the luminous flux of the 20 lamps

$$\ln L(t) = \theta + \lambda t + \epsilon,$$

where $L(t)$ is the observed luminous flux and the error term ϵ is assumed to follow a normal distribution. Due to the small error in the luminous flux readings, Tseng et al. (1995) directly predicted the lifetimes of the 20 lamps by the following formula:

$$\hat{t}_{ij} = \frac{1}{\hat{\lambda}_{ij}} * [\ln(0.6 * L(100)) - \hat{\theta}_{ij}].$$

where $\hat{\lambda}_{ij}$ and $\hat{\theta}_{ij}$ are the least squares estimates of λ and θ for the jth lamp at run i, respectively. Subsequently, based on the assumption that the lifetimes are lognormally distributed, Tseng et al. (1995) obtained the maximum likelihood estimates of the location and scale parameters $(\hat{\mu}_i, \hat{\sigma}_i^2)$ at run i by using directly the predicted lifetimes. The values of $(\hat{\mu}_i, \hat{\sigma}_i^2)$ and the predicted lifetimes of the 20 lamps are listed in Table 1. Since the 95% confidence intervals (CIs) of the effects of Factors B and C don't contain 0, Tseng et al. (1995) concluded that B and C are significant.

In fact, Weibull and lognormal distributions are much alike. In practical applications, they may fit the lifetime data well. However, their predictions may lead to a significant difference. That is, mis-discriminant between these two distributions may lead to serious bias. Fig. 1 shows the Weibull probability plots for the data of fluorescent lamps. The linear patterns in Fig. 1 indicate the appropriateness for fitting Weibull distribution to the data of fluorescent lamps. Thus, with respect to this experiment, a practical and interesting question is in the following:

Question: If the distributions of the failure times of fluorescent lamps are actually Weibull, not lognormal, then how "bad" will the precision of selecting the important factors be?

Due to the equivalence of lognormal (Weibull) and normal (extreme value) distributions, in the following sections, we will propose a systematic approach to address the issue based on the log lifetimes of fluorescent lamps.

3. Assumptions of a screening experiment and the measure of selection quality

3.1. Assumptions of a screening experiment

Assume that a resolution III design $L_r(2^m)$ with r runs and m factors is conducted. The experimental settings are summarized in Table 2. where the design points, r, is a multiple of 4, all the factors have two levels, and the number of factors is $m \leqslant r-1$. Besides, for $1 \leqslant i \leqslant r$, and $1 \leqslant h \leqslant m$,

$$c_{ih} = \begin{cases} -1 & \text{if } F_h \text{is evaluated at low level} \\ 1 & \text{if } F_h \text{is evaluated at high level.} \end{cases}$$

Table 1
The experimental setting, predicted lifetimes, estimated location-scale parameters $(\hat{\mu}_i, \hat{\sigma}_i^2)$, and the 95% confidence intervals of three main effects given in Tseng et al. (1995).

Factor run	A	B	C	Predicted lifetimes	$(\hat{\mu}_i, \hat{\sigma}_i^2)$
1	–	–	–	14762.98	
				16145.74	
				13429.86	(9.53, 0.1007)
				12941.79	
				12127.27	
2	–	+	+	26380.14	
				22860.88	
				24436.63	(10.4, 0.0970)
				20694.20	
				20468.02	
3	+	–	+	15028.19	
				15914.50	
				13708.69	(9.67, 0.0918)
				17285.64	
				17578.56	
4	+	+	–	20349.38	
				15000.83	
				17600.54	(9.81, 0.1056)
				19395.51	
				18746.54	
95% CI	(−0.1526, 0.0572)	(0.2144, 0.4242)	(0.0797, 0.2895)		