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A new method based on the dispersion of weights in data envelopment analysis

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Abstract

One of the drawbacks of the data envelopment analysis (DEA) is the problem of lack of discrimination among efficient decision making units (DMUs) and hence yielding many numbers of DMUs as efficient. The main purpose of this study is to overcome this inability. In the case in which the minimization of the coefficient of variation (CV) for input–output weights is added to the DEA model, more reasonable and more homogeneous input–output weights are obtained. For this new proposed model based on the CV it is observed that the number of efficient DMUs is reduced, improving the discrimination power. When this new approach is applied to two well-known examples in the literature, and a real-world data of OECD countries, it has been seen that the new model yielded a more balanced dispersion of input–output weights and reduced the number of efficient DMUs. In addition, the applicability of the new model is tested by a simulation study.

Keywords: Data envelopment analysis; Weight dispersion; Weight restriction; Coefficient of variation; Discrimination power

1. Introduction

Data envelopment analysis (DEA) is a fractional mathematical programming technique that has been developed by Charnes, Cooper, and Rhodes (1978). It is used to measure the productive efficiency of decision making units (DMUs) and evaluate their relative efficiency. This analysis determines the productivities of DMUs, specified as the ratio of the weighted sum of outputs to the weighted sum of inputs, compares them to each other and determines the most efficient DMU(s). DEA obtains the optimal weights for all inputs and outputs of each unit without imposing any constraints on these weights.

In DEA we sometimes encounter extreme values or zeroes in input and/or output weights for examined DMUs. In some cases we meet the unfitness of weights, i.e., a solution giving a big weight to variables with less importance or giving a small or zero weight to important variables. Especially in the zero cases, weights of input and/or output do not contribute to interpret the results of analysis. In the literature, various efforts have been done to overcome this problem. The problem of unrealistic weights in DEA has been tackled mainly by

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the techniques of weight restriction. Thompson, Singleton, Thrall, and Smith (1986, 1990) developed the *assurance region* approach to help choosing a best site for the location of high energy physics laboratory in Texas. The name of assurance region comes from the constraint which limits the region of weights to some special area. Charnes, Cooper, Huang, and Sun (1990) developed an approach that they called *cone ratio envelopment* in order to evaluate bank performances when unknown allowances for risk and similar factors needed to be taken into account. Wong and Beasley (1990) provide a weight restriction method by setting bounds on the proportions of individual inputs (or outputs) to total input (or output). The value efficiency developed by Halme, Joro, Korhonen, Salo, and Wallenius (1999) is an efficiency concept, which takes into consideration the decision maker's preferences. Li and Reeves (1999) have been developed multi-criteria data envelopment analysis (MCDEA) with the aid of one criterion efficiency evaluation methods. In a multi-criteria problem, it is generally impossible to find a solution that optimizes all criteria simultaneously. The Li and Reeves approach gives non-dominant (non-optimal) solutions. These non-dominant solutions can also be different to the preferences of decision maker. Recently, using the goal programming and DEA, Jahanshahloo, Memariani, Hosseinzadeh, and Shoja (2005) have been proposed a feasible interval of weights. This method uses the bounds on weights considered by decision maker. Because all of these methods incorporate additional constraints to the model they make harder to solve the problem and may cause to infeasibility.

In this study, a new method is developed for the betterment of dispersion of input–output weights based on the minimization of coefficient of variation (CV), and hence improving the discrimination power of the DEA method. For our new approach it is not necessary any a priori information involving human value judgement and it does not need any additional constraints on weights, as well. Moreover, it seems there is no unfeasibility problem for the solutions to our approach.

The paper is organized as follows. In Section 2, the basic DEA model and related concepts are given. In Section 3, the data envelopment analysis based on CV model (CVDEA) is presented and its formulation is explained. In Section 4, both the DEA and the CVDEA models are applied to two examples and their solutions are compared in respects of the number of efficient DMUs and the dispersion of input–output weights. In Section 5, the performances of the DEA and the CVDEA models are compared by a detailed simulation data. In Section 6, the DEA and the CVDEA models are applied to a real data set related to the OECD countries. Lastly, in Section 7, a summary of the research and its results are provided.

2. Data envelopment analysis

DEA evaluates the relative efficiency of homogeneous units by considering multiple inputs and outputs. Inputs can be resources used by a DMU and outputs can be products produced and/or performance measures of the DMU. The efficiency is defined as a ratio of the weighted sum of outputs to the weighted sum of inputs. DEA has been extensively used to compare the efficiencies of non-profit and profit organizations such as schools, hospitals, shops, bank branches and other environments in which there are relatively homogeneous DMUs (Baker and Talluri, 1997; Cooper et al., 2000).

Assuming that there are n DMUs each with m inputs and s outputs, the relative efficiency of a particular DMUo is obtained by solving the following fractional programming problem:

$$\begin{aligned} w_o = \max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} & \\ \text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, & \quad j = 1,2,\ldots,n, \\ u_r \geqslant 0, & \quad r = 1,2,\ldots,s, \\ v_i \geqslant 0, & \quad i = 1,2,\ldots,m, \end{aligned} \tag{1}$$