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Selecting the best statistical distribution with PROMETHEE and GAIA ☆

Alessio Ishizaka [a,*], Philippe Nemery [b]

[a] University of Portsmouth, Portsmouth Business School, Richmond Building, Portland Street, Portsmouth PO1 3DE, United Kingdom
[b] University of Portsmouth, Department of Mathematics, Lion Gate Building, Portsmouth PO1 3HF, United Kingdom

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A B S T R A C T

Three methods have previously been presented in *Computer and Industrial Engineering* for the selection of a statistical distribution to describe a data-set: the weighted sum model, the weighted multiplication model and data envelopment analysis. These are based on distinctive preset of parameters and result in three different rankings. In these approaches there is no interaction with the decision-maker (DM). This leads to the question: which method should a DM choose? In this paper, we adopt another approach where the DM is the central actor. Based on the multi-criteria decision aid methods, PROMETHEE and GAIA, we will show that different preference parameters (given by the DM) lead to different rankings. Finally, a group decision can be reached using its extension: PROMETHEE GDSS.

1. Introduction

A debate has arisen recently over the best method for selecting a probability distribution to represent a set of data (Ramanathan, 2005; Tofallis, 2008; Wang, Yam, & Zuo, 2004). Wang et al. (2004) have proposed a simple weighted additive model, whereas Ramanathan (2005) prefers data envelopment analysis and Tofallis (2008) supports a weighted multiplicative model. These three multi-criteria decision making methods produce a ranking (which may all be different) based on setting parameters distinctive to each method:

- The weighted additive model assumes linear indifference functions.
- The weighted multiplicative model has convex indifference curves, which favours compromises over extreme solutions.
- The data envelopment analysis (DEA) will, through a linear optimisation, choose weights which show each candidate under their best profile. Unlike the two other methods, it is not compensatory in the sense that bad scores may be ignored.

These three proposed methods have a prescriptive approach based on normative hypotheses, which are included in the method and do not depend neither on the data nor the type of problem. They aim to prescribe an optimal solution based on a rational model, established a priori to represent a simplified version of the reality. For example, in a weighted additive model, we assume that a 4% error is twice worse than a 2% error. In a multiplicative model, 4% is n power (where n is the weight of the criterion) worse than a 2% error. This is not always the case: nor an additive neither a multiplicative modelling would be appropriate if break-downs take place only above 3% error. The expertise of the decision-maker is therefore essential to model the problem. In the American school (or classical school) of Multi-Criteria Decision Making (MCDM), the weighted additive model have been improved to incorporate the expertise and preferences of the decision-maker, for example in MAUT and AHP. However, these methods still keep the normalisation problem of the weighted additive model (Triantaphyllou, 2001; Wang & Luo, 2009). In order to avoid the normalisation problem, outranking methods are used. They belongs to the French (or European) school and have also the advantage to prefer a constructive approach based on an interaction between the decision-maker and an analyst (the specialist in decision aid methods) (e.g. Vincke, 1992). These methods belong to the Multi-Criteria Decision Aid (MCDA) field. Because real decision problems are complex, fuzzy, unclear and not well specified, it is not possible to have completely stable and defined preference system in the mind of the decision-maker before even beginning the decision aiding process. It is only during the decision process and its interactions with the analyst, that the structure of the problem will become clear. This jointly constructed model must be a tool for looking, exploring, interpreting, debating and even arguing the problem (Roy, 2009; Tsoukiàs, 2008). Then, the parameters characterising the preference model are defined. Several sets of parameters may be accepted or investigated in order to evaluate the impact of each one on the produced results. In the American

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* Corresponding author.
E-mail addresses: Alessio.Ishizaka@port.ac.uk (A. Ishizaka), pnemery@gmail.com (P. Nemery).

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conception, these parameters are predefined at the exception of the weights of the criteria and sometimes the utility functions. The French school certainly requires a longer process needing several revisions but the decision-maker will better understand the results and potentially explain and defend them (Roy, 1996).

PROMETHEE has already been used successfully in several cases. Behzadian, Kazemzadeh, Albadvi, and Aghdasi (2010) enumerated 195 papers, from its conception until 2008, where PROMETHEE is applied in environment management (47 papers), business and financial management (25), hydrology and water management (28), chemistry (24), logistics and transportation (19), manufacturing and assembly (19), energy management (17), social (7), design (2), agriculture (2), education (2), sports (1), information technology (1) and medicine (1). Recently, PROMETHEE has been used in water management (Kodikara, Perera, & Kularathna, 2010; Silva, Morais, & Almeida, 2010), banking (Doumpos & Zopounidis, 2010), energy management (Ghafghazi, Sowlati, Sokhansanj, & Melin, 2010; Oberschmidt, Geldermann, Ludwig, & Schmehl, 2010), manufacturing and assembly (Kwak & Kim, 2009; Saidi Mehrabad & Anvari, 2010; Tuzkaya, Gülsün, Kahraman, & Özgen, 2010; Venkata Rao & Patel, 2010; Zhu, Xu, Chen, & Li, 2010), logistics and transportation (Lanza & Ude, 2010; Safaei Mohamadabadi, Tichkowsky, & Kumar, 2009; Semaan & Zayed, 2010), quality (Nikolic, Jovanovic, Mihajlovic, & Zivkovic, 2009), chemistry (Cornelissen et al., 2009; Ni, Lai, Brandes, & Kokot, 2009), maritime commerce (Castillo-Manzano, Castro-Nuño, Laxe, López-Valpuesta, & Teresa Arévalo-Quijada, 2009), strategy (Ghazinoory, Divsalar, & Soofi, 2009), project management (Halouani, Chabchoub, & Martel, 2009), construction (Castillo-Manzano et al., 2009; Frenette, Beauregard, Abi-Zeid, Derome, & Salenikovich, 2010), urban development (Juan, Roper, Castro-Lacouture, & Kim, 2010), location analysis (Luk, Fernandes, & Kumar, 2010), environment (Nikolić et al., 2010; Soltanmohammadi, Osanloo, & Aghajani Bazzazi, 2009; Zhang, Achari, & Pei, 2010; Zhang, Kluck, & Achari, 2009), safety (Ramzan, Naveed, Feroze, & Witt, 2009) and e-commerce (Andreopoulou, Kokkinakis, & Koutroumanidis, 2009). PROMETHEE method is on the basis of two sorting methods: Promsort (Araz & Ozkarahan, 2007) and FlowSort (Nemery & Lamboray, 2008).

As selecting a statistical distribution is a complex, fuzzy and unclear problem, which needs interactions with the decision-maker, PROMETHEE method complemented by GAIA (Brans & Mareschal, 1994) is appropriate. This method permits easily the modelling of this decision problem according to the preferences of the decision-maker. Moreover, the developed software Smart Picker Pro (downloadable at www.smart-picker.com) supporting PROMETHEE and GAI has a user-friendly graphical interface, which facilitates the interaction.

In this paper, we first discuss the methods proposed in the previous papers. In the next section, we present three different scenarios modelled with PROMETHEE and GAIA, which lead to different results. Then, we introduce PROMETHEE Group Decision Support System (GDSS) in order to incorporate the view of several decision-makers. Finally, we conclude the paper with a discussion on the advantages of the proposed approach.

2. Review of proposed methods

2.1. Description of the problem

The problem consists in the selection of a probability distribution to represent a set of data based on the following criteria:

- $D_{\max}$: the Kolmogorov–Smirnov statistic test,
- δ_f: the average deviation between the theoretical probability distribution function and the empirical one,
- δ_F: the average deviation between the theoretical cumulative distribution function and the empirical one,
- D: the deviation in skewness and kurtosis,
- E: a subjective score obtained from a group of experts in the field of study and statistics on the user friendliness of the distribution and the frequency of its use in the field, and the fitness of properties and characteristics of the distribution to the sampled data.

Wang et al. (2004) provides the following data from an engineering problem involving machine tools (Table 1).

It is not the scope of this paper to discuss these criteria. If appropriate, other criteria could be used. The focus here will be on the selection method.

2.2. Weighted sum approach

In a weighted sum approach, performances of the actions on each criterion are simply weighted according to the importance of the criteria and then added. When measures are not commensurate (like in Table 1), a standardisation is necessary. Wang et al. (2004) proposes the transformation function:

$$r(v) = 1/(1 + cv^2) \tag{1}$$

where c is a positive constant.

Ramanathan (2005) gives an example where different values of the constant c lead to different final rankings. Tofallis (2008) adds that there are several ways of standardising: z-transformation, dividing the performances by their sum, dividing the performances by the highest performance, etc. and all have merits and drawbacks but may lead to different results, without knowing the causal effect. To overcome this problem, approaches which do not require any standardisation have been proposed.

The weighted sum approach is a very simple model, where the utility functions are linear positive. For example, a deviation of 4% is always twice worse than a deviation of 2% and four times worse than a deviation of 1%. Furthermore, the incomparability is not considered but assimilated to indifference in this model as well as in the weighted multiplicative model. Two alternatives reaching the same score are said to be indifferent, even if the way they obtained this score is very different and therefore incomparable (Vincke, 1992).

2.3. Data envelopment analysis

Ramanathan (2005) proposes the use of the data envelopment analysis (DEA), which is an often used ranking technique (Adler, Friedman, & Sinuany-Stern, 2002; Mannino, Hong, & Choi, 2008; Serrano-Cinca, Fuertes-Callén, & Mar-Molinero, 2005; Sueyoshi & Goto, 2009a, 2009b) and does not require any normalisation and even any subjective input (e.g. weight of the criteria) from the user. However, Tofallis (2008) points out three major problems with DEA:

- DEA is not designed for selecting a single winner, it indicates only the non-dominated solutions.
- DEA may completely ignore the weakness of some candidates.

Table 1
Performances of each distribution.

Distribution	$D_{\max}$	δ_F	δ_f	D	E
Beta	0.144612	0.000845215	0.0404891	2.79466	0.22
Gamma	0.09821	0.000431302	0.0088562	0.66035	0.22
Weibull	0.056581	0.000397474	0.0110291	1.288	0.18
Lognormal	0.10316	0.000660129	0.0205139	3.15615	0.18
Normal	0.404622	0.00172495	0.23522	4.06326	0.1
Extreme-value	0.176833	0.00155293	0.0402712	1.03315	0.1