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Economic design of two-stage non-central chi-square charts for dependent variables ☆

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ABSTRACT

Non-central chi-square charts are more effective than the joint $\bar{X}$ and R charts in detecting small mean shifts or variance changes of a performance variable. However, the cost may be high to monitor a primary quality characteristic, such as the weight of each bag in a cement filling process. It is more economical to monitor a surrogate variable, for example, the milliampere of the load cell. When the correlation of the performance variable of surrogate variable exists, this article proposes a two-stage charting design to monitor either the performance variable or its surrogate variable in an alternating fashion rather than monitoring the performance variable alone. The proposed method simplifies process monitoring when users only concern about whether a process is in control or not. The application of the proposed method and the advantages of the proposed chart over the existing methods are presented through an example. Numerical results show that the proposed chart is insensitive on the correlation of the performance variable and surrogate variable even when the historical information on the correlation coefficient is not very accurate.

1. Introduction

Statistical process control (SPC) procedures for variables have become a very important tool in the manufacturing and process industries for detecting changes in the quality of products. Traditionally, Shewhart $\bar{X}$ and R charts were developed for monitoring the process mean and variance of a quality characteristic respectively. In some cases, it may be beneficial to monitor quality of products by using both primary variable, or called performance variable, and a surrogate variable in an alternating fashion for cost reduction for implementing SPC. For example, the quantity of liquid in each bottle, denoted by X, is the primary quality characteristic in a filling process. Charts were developed traditionally to monitor the quality of the filling process. The distance between the top of the bottle and the surface of the liquid inside the bottle, denoted by Y, is highly correlated with X. An electronic vision device fixed on the conveyer can be used to measure Y, while X is measured in a laboratory. Hence, Y is easier and cheaper to measure during production than measuring X in a laboratory. A two-stage charting approach by switching the monitoring of either Y or X is more economical than only using the charting on X. In practice, practitioners may monitor the process by using the surrogate variable only. However, the use of a surrogate variable alone for process monitoring may increase its false alarm rate.

Lee and Kwon (1999) proposed an economic design of a two-stage chart which uses a highly correlated surrogate variable together with its performance variable. The process is monitored by the surrogate variable until it provides an out-of-control signal, then by the performance variable directly until additional out-of-control signals are triggered or in-control signals are maintained for a pre-specified amount of time. Both primary and surrogate variables are used in an alternating fashion. Following the switch mechanism between the two-stage charts proposed by Lee and Kwon (1999) and Costa and De Magalhães (2005) considered an economic design of a two-stage $\bar{X}$ chart based on a Markov chain approach. The aforementioned approaches only focus on monitoring the process means.

In recent years, many researchers have paid attention to the developments of joint charts for monitoring the process mean and variance. Costa and Rahim (2000) and Rahim and Costa (2000) proposed economic designs for joint $\bar{X}$ and R charts. Gan (1995) suggested a joint EWMA chart. Albin, Kang, and Sheha (1997) studied an individual chart and an EWMA chart. Chen, Cheng, and Xie (2001) combined two EWMA charts into one chart and concluded that the new EWMA chart works well in detecting both increases and decreases in the process mean or variance. Costa (1998, 1999) and De Magalhães and Moura Neto (2005) studied an

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adaptive design of joint $\bar{X}$ and R charts with variable parameters of the charts. Reynolds and Stoumbos (2001) investigated three joint individual charts for monitoring the process mean and variance of a normal quality variable with variable sampling intervals. Costa and Rahim (2004) developed a non-central chi-square (NCS) chart to monitor the process mean and variability simultaneously. In addition, Costa and De Magalhães (2007) proposed an adaptive chart for monitoring the process mean and variance. However, these works did not pay attention to the development of control charts to monitor the process mean and variance of the primary variable and its surrogate variable simultaneously. The benefits of this kind of control charts include quick detection of process changes and simplification in process surveillance.

In this paper, we propose an economically designed two-stage NCS chart approach for dependent variables to monitor both process mean and variance. During the first stage of the proposed method, the process mean and variance of the surrogate variable are to be monitored by a NCS chart, named NCS-*Y* chart. Once the NCS-*Y* chart provides an out-of-control signal, another NCS chart is used to monitor the mean and variance of the primary or performance variable. This second NCS chart is named NCS-*X* chart with central region, warning region and action region. If the NCS-*X* statistic falls into the central region, the process surveillance returns to the NCS-*Y* chart. Otherwise, the process surveillance remains under the NCS-*X* chart until an NCS *X* statistic falls into the action region, at which time searching for an assignable cause is undertaken.

The rest of this paper is organized as follows: In Section 2, we review the NCS chart proposed by Costa and Rahim (2000). The economic design of two-stage NCS charts is established using Markov chain approach in Section 3. An intensive numerical study is conducted and an example is used in Section 4 to demonstrate the economic advantages of the proposed method. Finally, concluding remarks are given in Section 5.

2. Literature review

Costa and Rahim (2004) proposed a NCS chart to monitor the process mean and variance simultaneously. They assumed that the chart is employed to monitor a process whose quality characteristic of interest (say, X) is normally distributed with mean, μ, and standard deviation, σ. It is assumed that μ_0 and σ_0 are the in-control values of process mean μ and standard deviation σ, respectively. After an assignable cause is introduced, the process mean shifts to $\mu_1 = \mu_0 \pm \delta\sigma_0$, and the process standard deviation shifts to $\sigma_0 = \gamma\sigma_0$, where $\delta \neq 0$ and $\gamma \neq 1$. In practice, quality engineers are more interested in detecting process changes when $\gamma > 1$. The case of $\gamma > 1$ indicates a deterioration of manufacturing quality. Let x_j be the value of X, measured on the jth item of the sample. The NCS chart is constructed based on the plotting statistic

$$G = \sum_{j=1}^{n} (x_j - \mu_0 + \xi\sigma_0)^2,$$

If $\bar{X} > \mu_0$, $\xi = d$, otherwise $\xi = -d$, where d is a positive constant. In general, larger values of d are better for detecting mean-only shifts, i.e., with process parameters μ_1 and σ_0, and worse for detecting variance-only increases such as the case of process parameters μ_0 and σ_1.

If the process is in control, G/σ_0^2 has a NCS distribution with n degrees of freedom and a non-centrality parameter, $\lambda_0 = nd^2$, denoted by $G/\sigma_0^2 \sim \chi_n^2(\lambda_0)$. Accordingly, the false alarm rate of this chart is given by

$$\alpha = \Pr[G > K_{chi}\sigma_0^2 | G/\sigma_0^2 \sim \chi_n^2(\lambda_0)]$$

where K_{chi} is the factor used in determining the upper control limit for the NCS chart. During the out-of-control period, G/σ_1^2 is distributed as a NCS distribution with n degrees of freedom and a non-centrality parameter $\lambda_1 = n(\delta + \xi)^2/\gamma^2$. Accordingly, the power of the chart is given by

$$1 - \beta = \Pr[G > K_{chi}\sigma_0^2 | G/\sigma_1^2 \sim \chi_n^2(\lambda_1)]$$

3. The economic design of two-stage NCS charts

In some applications, the cost is high for collecting measurements of performance variable X but is low for collecting those of surrogate variable Y, where X and Y are highly correlated. Assume that measurements of (X_i, Y_i), $i = 1, 2, \ldots$ are collected from the process according to the following modeling relationship:

$$X_i = \xi_i + \delta_i,$$
$$Y_i = \beta_0 + \beta_1\xi_i + \varepsilon_i \cdot i = 1, 2, \ldots$$

where δ_i's are independent normal variables with mean 0 and variance σ_δ^2; ε_i's are random errors that are normally distributed with mean 0 and variance σ_ε^2; and ξ_i's are independent random variables that follow a normal distribution with mean ξ and variance σ_ξ^2. Moreover, δ_i's are independent of ε_i's, and ξ_i's are independent of both δ_i's and ε_i's. Fuller (1987) showed that the random vector of (X_i, Y_i) have a bivariate normal distribution with mean vector

$$(\mu_x, \mu_y)^T = (\xi, \beta_0 + \beta_1\xi)$$

and covariance matrix

$$\begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix} = \begin{pmatrix} \sigma_\delta^2 + \sigma_\xi^2 & \beta_1\sigma_\xi^2 \\ \beta_1\sigma_\xi^2 & \beta_1^2\sigma_\xi^2 + \sigma_\varepsilon^2 \end{pmatrix}.$$

In this paper, we use the NCS-*Y* chart and NCS-*X* chart in an alternating fashion to monitor a process which is subject to a single assignable cause. The process surveillance starts by monitoring Y until the NCS-Y chart provides an out-of-control signal, then it is switched to the monitoring of X via the NCS-*X* chart until it provides an additional out-of-control signal or in-control signals have reached a pre-specified amount of time.

It is assumed that the assignable cause occurs according to a Poisson process with an intensity of λ occurrences per unit time. After the assignable cause occurrence, the process mean shifts to $\mu_{x1} = \mu_{x0} + \delta_x\sigma_{x0}$ from μ_{x0} and the process standard deviation shifts to $\sigma_{x1} = \gamma_x\sigma_{x0}$ from σ_{x0}. The process mean of Y shifts to $\mu_{y1} = \mu_{y0} \pm \beta_1\delta_x\sigma_{y0} = \mu_{y0} \pm \delta_y\sigma_{y0}$ from μ_{y0}, where $\delta_y = \beta_1\delta_x$; and the standard deviation shifts to $\delta_{y1} = \gamma_y\sigma_{y0}$ from σ_{y0}.

Let n_y, h_y, d_y and n_x, h_x, d_x denote the sample size, the length of sampling interval and constant d for the NCS-*Y* chart and the NCS-*X* chart, respectively. Let k_x and ω_x be the factors of action and warning limits, respectively for the NCS-*X* chart; and let k_y be factor of action limit for the NCS-*Y* chart. Let $\chi_{n_y}^2(\lambda_y) = G_y/\sigma_{y0}^2$ and $\chi_{n_x}^2(\lambda_x) = G_x/\sigma_{x0}^2$. The design procedure of two-stage charts can be summarized as follows:

Step 1: Take a sample of size n_y after an interval of h_y time units.
Step 2: If $\chi_{n_y}^2(\lambda_y) < k_y$, go to Step 1. Otherwise, go to Step 3.
Step 3: Take a sample of size n_x after an interval of h_x time units and go to Step 4.
Step 4: If $\chi_{n_x}^2(\lambda_x) < \omega_x$, go to Step 1. Otherwise, go to Step 3 until $\chi_{n_x}^2(\lambda_x) < k_x$ then go to Step 5.
Step 5: Stop process monitoring and search for the assignable cause. If the signal is a false alarm, go to Step 1. Otherwise, eliminate the assignable cause, and then go to Step 1.

Because the sampling intervals are not constant in the proposed charting process, one of the commonly used performance indicator in assessing charting property, the average run length (ARL), is improper to be used as the performance indicator for the proposed