Computers & Industrial Engineering 61 (2011) 1149–1159

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Control of discrete event systems with respect to strict duration: Supervision of an industrial manufacturing plant ☆

Abdourrahmane M. Atto [a,*], Claude Martinez [b], Saïd Amari [c]

[a] LISTIC, Université de Savoie, Polytech Annecy-Chambéry, France
[b] IRCCyN UMR CNRS 6597, LUNAM Université, Université de Nantes, Nantes, France
[c] LURPA, ENS Cachan, France

ARTICLE INFO

Article history:
Received 25 February 2008
Received in revised form 15 October 2010
Accepted 11 July 2011
Available online 26 July 2011

Keywords:
Discrete event systems
(max,+) Algebra
Dioids
Time constraints
Supervisor
Control

ABSTRACT

In this paper, we propose a (max,+)-based method for the supervision of discrete event systems subject to tight time constraints. Systems under consideration are those modeled as timed event graphs and represented with linear (max,+) state equations. The supervision is addressed by looking for solutions of constrained state equations associated with timed event graph models. These constrained state equations are derived by reducing duration constraints to elementary constraints whose contributions are injected in the system's state equations. An example for supervisor synthesis is given for an industrial manufacturing plant subject to a strict temporal constraint, the thermal treatment of rubber parts for the automotive industries. Supervisors are calculated and classified according to their performance, considering their impact on the production throughput.

1. Introduction

Discrete Event Systems (DES) are of great interest in research activities dedicated to industrial production systems. Many approaches have been proposed for the analysis of DES these last few decades (see Cassandras & Lafortune, 1992 among others). Some are related to computational simulations (Law & Kelton, 1991), and others are based on the (max,+)-algebra (Baccelli, Cohen, Olsder, & Quadrat, 1992; Gaubert, 1992). Under some assumptions, DES can be modeled as Timed Event Graphs (TEGs) (Gaubert, 1992; Murata, 1989) and thus, the analysis of the system can be described with linear equations in (max,+)-algebra.

This work concerns the supervision of DES, modeled as TEGs, and assumed to respect strict temporal constraints for specific processing. The supervision is aimed at guaranteeing the respect of temporal constraints without impacting significantly the dynamic behavior of the system. Similar problems of meeting time constraints have been recently addressed via different approaches (Amari, Loiseau, & Demongodin, 2005; Houssin, Lahaye, & Boimond, 2007; Kim & Lee, 2003; Ouerghi & Hardouin, 2006; Spacek, Manier, & Moudni, 1999). We propose solutions based on a constrained (max,+) state equation for the TEG model of the DES. Constrained state equations are obtained by reducing temporal constraints to elementary constraints and by injecting contributions of these elementary constraints in the state equation of the TEG models. These elementary constraint equations derive from a simplified representation of the TEG under consideration, representation which consists in decomposing a place with a number m of tokens into m places, each one containing only one token.

The method proposed in this paper is used to synthesize supervisors for an industrial manufacturing plant subject to a strict temporal constraint. Supervisors are calculated and classified according to their performance. The performance evaluation is of great interest in the literature on the topic of (max,+) algebra (Baccelli et al., 1992; Cohen, Moller, Quadrat, & Viot, 1985; Gaubert, 1992). The performance of a supervisor is measured according to the maximum throughput of the plant. According to this particular performance measure, we can classify the supervisors between those which slow down the production throughput and those which preserve this production rate. The cycle time of such a plant modeled as a TEG corresponds to the eigenvalue of the matrix associated with its graph (Baccelli et al., 1992; Cohen, Dubois, Quadrat, & Viot, 1983; Gaubert, 1995); the production throughput is the inverse of the cycle time.

This work is organized as follows. Section 2 recalls basics on (max,+) theory and provides a description of the dynamic behavior of TEGs according to linear (max,+) models. Section 3 first presents

☆ This manuscript was processed by Area Editor Gursel A. Suer.

* Corresponding author.
E-mail addresses: am.atto@ieee.org (A.M. Atto), claude.martinez@univ-nantes.fr (C. Martinez), said.amari@lurpa.ens-cachan.fr (S. Amari).

0360-8352/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cie.2011.07.004

the integration of temporal constraints in this linear model. Then, the supervision problem is addressed and methods are provided for supervisor synthesis. A first case study is discussed for the supervision of a constrained system. A second example is provided for the control a single armed robot in a cluster-tool for the semiconductor industry. Section 4 discusses the supervision of an industrial plant. The supervision is aimed at guaranteeing the respect of a strict duration constraint for a heating process without impacting significantly the production rate of the manufacturing unit. In this section, we show that the supervision can be performed thanks to the analytical technique described in Section 3 and provide supervisors that allow for preserving the production rate of the industrial plant. Finally, Section 5 gives a conclusion and addresses perspectives to extend this work.

2. TEG and linear (max,+) models

2.1. (max,+) algebra

This section briefly recalls the fundamentals of (max,+) algebra, which is largely used for the analysis of DES. Further details about this theory can be found in Gondran and Minoux (1977), Cohen, Dubois, Quadrat, and Viot (1985), Gaubert (1992), Cohen (1994), Libeaut (1996) among others. Some specific results that are essentials to state on the existence of a solution to the problem tackled here are therefore given at the end of the section. In what follows, $\mathcal{D}$ denotes a set.

Definition 1 (*Monoid*). A monoid is an algebraic set with an associative internal operation and an identity element.

Definition 2 (*Semiring*). $(\mathcal{D}, \oplus, \otimes)$ is a semiring if:

- $(\mathcal{D}, \oplus)$ is a commutative monoid. Its identity element is denoted by ϵ (null element).
- $(\mathcal{D}, \otimes)$ is a monoid. Its identity element is denoted by e (unit element).
- Multiplication $\otimes$ distributes over addition and ϵ annihilates $\mathcal{D}$ (every $x \in \mathcal{D}$ is such that $x \otimes \epsilon = \epsilon \otimes x = \epsilon$).

Definition 3 (*Dioid*). A dioid $(\mathcal{D}, \oplus, \otimes)$ is an idempotent semiring (every $x \in \mathcal{D}$ is such that $x \oplus x = x$).

Hereafter, the product $a \otimes b$ will be denoted $a.b$ or ab when there is no possible confusion.

Example 1 (*Examples of dioids*).

- Let $\mathbb{R}$ be the set of real numbers. $(\mathbb{R} \cup \{-\infty\}, \max, +)$ is a commutative dioid for which $\epsilon = -\infty$ and $e = 0$. This dioid is denoted by $\mathbb{R}_{\max}$ and is called (max,+) algebra.
- Let $(\mathcal{D}, \oplus, \otimes)$ be a dioid and $\mathcal{D}^{n\times n}$ the set of square matrices of order n over $\mathcal{D}$. $(\mathcal{D}^{n\times n}, \oplus, \otimes)$ is a dioid called a matrix dioid. The sum and the matrix product are defined as follows: if $A = (A_{ij})$, $B = (B_{ij})$, then $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$ and $(A \otimes B)_{ij} = \oplus_{k=1}^{n} A_{ik} \otimes B_{kj}$. The null element of the matrix dioid is the matrix composed of ϵ. The unit matrix is the matrix with e on the main diagonal and ϵ elsewhere.

Definition 4 (*Moduloid*). Let $\mathcal{D}$ be a dioid. A moduloid $\mathcal{M}$ over $\mathcal{D}$ is a monoid $(\mathcal{M}, \oplus)$, endowed with an external operation "." $\mathcal{D} \times \mathcal{M} \to \mathcal{M}$, such that for all $\lambda, \mu \in \mathcal{D}$ and for all $u, v \in \mathcal{M}$:

1. $(\lambda \oplus \mu).u = \lambda.u \oplus \mu.v$
2. $(\lambda.\mu).u = \lambda.(\mu.u)$
3. $\lambda(u \oplus v) = \lambda.u \oplus \lambda.v$
4. $\epsilon.u = \epsilon$
5. $e.u = u$

Example 2 (*Examples of moduloids*). The set $\mathcal{D}^n$ of n dimensional vectors over a dioid $\mathcal{D}$ is a moduloid over $\mathcal{D}$. In the same way, the set of $\mathcal{D}^{n\times m}$ matrices over a dioid $\mathcal{D}$ is a moduloid over $\mathcal{D}$.

In the rest of the paper (especially in Section 3), we consider the moduloid $\mathbb{R}_{\max}^{n\times m}$ defined over the dioid $\mathbb{R}_{\max}$.

Let $(\mathcal{D}, \oplus, \otimes)$ be a dioid. The idempotency of the operation $\oplus$ induces over $\mathcal{D}$, an order structure denoted $\preceq$ and defined by: $x \preceq y \Leftrightarrow x \oplus y = y$. This order relation is compatible with the operations $\oplus$ and $\otimes$ (proof in Baccelli et al. (1992)). In (max,+) algebra, this order coincides with usual order $\leqslant$. The lower ($\wedge$) and upper ($\vee$) bounds are defined by: $x \preceq y \Leftrightarrow x \oplus y = y \Leftrightarrow x \wedge y = x \Leftrightarrow x \vee y = y = x \oplus y$.

Definition 5 (*Completeness in dioids*). A dioid $(\mathcal{D}, \oplus, \otimes)$ is complete if

$$\forall c \in \mathcal{D}, \forall A \subseteq \mathcal{D}, \quad c \otimes (\bigoplus_{x\in A} x) = \bigoplus_{x\in A} c \otimes x,$$

that is if it is closed for infinite sums and if the operation $\otimes$ distributes over infinite sums.

A complete dioid $\mathcal{D}$ is upper bounded by an element, denoted T, defined by:

$$T = \oplus_{x\in\mathcal{D}} x.$$

This element annihilates $(\mathcal{D}, \oplus)$, that is $T \oplus x = T$ for all $x \in \mathcal{D}$, and this element also verifies: $T \otimes \epsilon = \epsilon$.

Note that dioid $\mathbb{R}_{\max}$ is not complete. $\mathbb{R}_{\max} \cup \{+\infty\}$ with the convention: $(-\infty) \otimes (+\infty) = (+\infty) \otimes (-\infty) = -\infty$ is a complete dioid denoted by $\overline{\mathbb{R}}_{\max}$.

Theorem 1. *Let $\mathcal{D}$ be a complete dioid. The least solution in x of $x \succeq ax \oplus b$ is $x = a^*b$, where $a^* = \oplus_{k\geqslant 0} a^k$ is the Kleene star of a.*

The proof of this theorem is given in Gaubert (1992).

The problem of multivariable control tackled in this article involves solving an inequality of the form $A \otimes x \geqslant B \otimes x$. In this problem, it is often usefull to address the existence of solutions that makes synthesis of controller possible. In this respect, the following lemma is actually a specific case of the results presented in Allamigeon, Gaubert, and Goubault (2010). Earlier works on resolution of this type of inequality can be found in Hegedüs and Butkovič (1984), Cuninghame-Green and Butkovič (2003), Butkovič, Schneider, and Sergeev (2007), in Walkup (1995) and in Cechlárová (2005).

Lemma 1. *Let $v, u \in \overline{\mathbb{R}}_{\max}^{1\times n}$ be given row vectors and $x \in \overline{\mathbb{R}}_{\max}^{n}$. The inequality $v \otimes x \geqslant u \otimes x$ admits a non-trivial solution if and only if there exists $k \in \{1, 2, \ldots, n\}$ such that $|v_k \geqslant u_k$.*

Let us call such index k a critical index. Let K_i be the set containing all critical indices in a row i of a matrix H. Cechlárová gives in Cechlárová (2005) a necessary and sufficient condition for the existence of a solution for equation of the form $H \otimes x = Q \otimes x$ with $H \geqslant Q$. This condition is expressed in the following theorem. The proof of this theorem is given in Cechlárová (2005). In this theorem, $\bar{p}$ denotes the set of row indices $\{1, 2, \ldots, p\}$ and $\bar{n}$ is the set of column indices $\{1, 2, \ldots, m\}$ of matrices H and Q.

Theorem 2. *A system $H \otimes x = Q \otimes x$ with $H \in \overline{\mathbb{R}}_{\max}^{p\times n}$, $H \geqslant Q$, is soluble if and only if*