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# An extended GRA method for MCDM with interval-valued triangular fuzzy assessments and unknown weights ☆

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ABSTRACT

Multiple criteria decision making (MCDM) is the process of ranking the feasible alternatives and selecting the best one by considering multiple criteria. Owing to the complexity, fuzziness and uncertainties of the objective things, the criterion values often take the form of linguistic variables, which can be expressed in interval-valued triangular fuzzy numbers. The purpose of this paper is to develop an extended grey relational analysis (GRA) method for solving MCDM problems with interval-valued triangular fuzzy numbers and unknown information on criterion weights. In order to determine the criterion weights, some optimization models based on the basic idea of traditional GRA method are established. Then, calculation steps of extended GRA method for MCDM are given. Finally, a numerical example is shown to verify the developed method and to demonstrate its practicality and feasibility.



## 1. Introduction

Multiple criteria decision making (MCDM) problem is a well-known branch of decision theory. It has been found in real life decision situations (Zhang, Fan, & Liu, 2010; Ho, Xu, & Dey, 2010; Cheng, 2008; Chang & Wang, 2009; Yu & Hu, 2010). A MCDM problem consists of ranking the feasible alternatives and selecting the most desirable one(s) by considering multiple criteria, which are frequently in conflict with each other. In the real world, due to the complexity, fuzziness and uncertainties of the objective things, the criteria involved in the decision making problem may not be appropriate to express them by exact numerical values. It is more suitable to describe them by means of linguistic variables. Hence, linguistic information has frequently been applied to MCDM problems.

Since linguistic variables are not directly mathematically operable, to cope with this difficulty, each linguistic variable is associated with a fuzzy number characterizing the meaning of each generic verbal term (Chou & Chang, 2008). In some existing literatures (Xu & Chen, 2007; Tuzkaya, Gülsün, Kahraman, & Özgen, 2010; Mahdavi, Mahdavi-Amiri, Heidarzade, & Nourifar, 2008; Wang & Lee, 2007; Wang & Chen, 2008; Chen, Wang, Chen, & Lee, 2010), linguistic variables are converted to triangular fuzzy numbers in the decision making process. The reason (Lam, Tao, & Lam, 2010) of using triangular fuzzy number is because of its intuitive, easy to use, computational simplicity, useful in promoting representation and information processing in a fuzzy environment. However, in some actual decision making process, it is difficult to determine the specific values for lower and upper bounds of triangular fuzzy number. If the values' range is relatively easy to determine, then it is appropriate to define lower and upper bounds values as an interval. Thus, it can not only avoid the loss of information, but also express the decision making information more precisely. Therefore, linguistic variables were converted to interval-valued triangular fuzzy numbers in some literatures. Ashtiani, Haghighirad, Makui, and Montazer (2009) presented a fuzzy TOPSIS method for handing MCDM problems, in which all the information on criterion values and criterion weights is represented as linguistic variables, expressed in interval-valued triangular fuzzy numbers. Vahdani, Hadipour, Sadaghiani, and Amiri (2010) considered the same problem and developed a fuzzy VIKOR method. The above two literatures mainly focus on the group MCDM problems with known information on criterion weights under interval-valued triangular fuzzy environment. However, because of time pressure and the expert's limited expertise about the problem domain, the information on criterion weights in the process of MCDM is sometimes completely unknown. The situation leads to the problem how to derive the criterion weights, which is an interesting and important research topic. Therefore, based on the above two

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literatures, this paper investigates the MCDM problem with unknown weights.

Grey relational analysis (GRA) (Deng, 1989; Deng, 2002) is a helpful tool in MCDM problems that was originally proposed by Deng. It has been successfully applied in solving a variety of MCDM problems. GRA is an impact evaluation model that can measure the correlation between series and belongs to the category of the data analytic method or geometric method. Usually, researchers will set the target series based on the objective of the studied problem as the reference series. Hence, the purpose of grey relational analysis method is to measure the relation between the reference series and comparison series.

In this paper, we propose an extended fuzzy GRA method to solve MCDM problems, in which the criterion values are in the form of linguistic variables expressed in interval-valued triangular fuzzy numbers and the information on criterion weights is unknown. In order to determine the criterion weights, some optimization models based on the basic idea of traditional GRA are established. Then, calculation steps of extended GRA method for MCDM with interval-valued triangular fuzzy assessments are given to rank the alternatives and select the desirable one. To do that, the rest of this paper is organized as follows. In Section 2, we briefly introduce the GRA method. Section 3 illustrates triangular fuzzy sets. Section 4 describes a developed GRA method to solve interval-valued triangular fuzzy MCDM problems with unknown weights. Section 5 investigates a numerical example including an application to select a system analysis engineer for a software company to illustrate the applicability of the proposed method. Finally, the conclusion is given in Section 6.

## 2. Grey relational analysis method

Suppose a multiple criteria decision making problem having $m$ non-inferior alternatives $A_1, A_2, \ldots, A_m$ and $n$ criteria $C_1, C_2, \ldots, C_n$. Each alternative is evaluated with respect to the $n$ criteria. All the evaluate values/ratings are assigned to alternatives with respect to decision matrix denoted by $X(=(x_{ij})_{m\times n})$.

The GRA procedure consists of the following steps:

Step 1. Calculate the normalized decision matrix. The normalized value $r_{ij}$ is calculated as:

$$r_{ij} = \frac{x_{ij}}{\max(x_{ij})}, \quad i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n, \quad \text{for } j \in I, \tag{1}$$

$$r_{ij} = \frac{\min(x_{ij})}{x_{ij}}, \quad i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n, \quad \text{for } j \in J. \tag{2}$$

where $I$ is the set of benefit criteria, and $J$ is the set of cost criteria.

Step 2. Determine the reference series $R_0$.

$$R_0 = \{r_{01}, r_{02}, \ldots, r_{0n}\}. \tag{3}$$

where $r_{0j} = max_j r_{ij}$, $j = 1, 2, \ldots, n$.

Step 3. Establish the distance matrix. The distance $\delta_{ij}$ between the reference value and each comparison value is given as

$$\delta_{ij} = r_{0j} - r_{ij}. \tag{4}$$

Then the distance matrix $\Delta$ can be obtained as:

$$\Delta = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ \vdots & \vdots & & \vdots \\ \delta_{m1} & \delta_{m2} & \cdots & \delta_{mn} \end{bmatrix}. \tag{5}$$

Step 4. Calculate the grey relational coefficient. The grey relational coefficient $\xi_{ij}$ is defined as:

$$\xi_{ij} = \frac{\delta_{min} + \zeta\delta_{max}}{\delta_{ij} + \zeta\delta_{max}}, \quad i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n. \tag{6}$$

where $\delta_{max}$ and $\delta_{min}$ are the maximum and minimum of $\delta_{ij}(i = 1,\ldots,m; j = 1,\ldots,n)$, respectively, and $\zeta$ is the distinguishing coefficient between 0 and 1. Usually, suppose that $\zeta$ is 0.5.

Step 5. Estimate the grey relational grade $\gamma_i$ by the relation

$$\gamma_i = \sum_{j=1}^{n} w_j \xi_{ij}, i = 1,2,\ldots,m. \tag{7}$$

where $w_j$ is the weight of the $j$th criterion, and $w_j \geqslant 0, \sum_{j=1}^{n} w_j = 1$.

Step 6. Rank the alternatives in accordance with the value of grey relational grade, the bigger the value $\gamma_i$, the better the alternative $A_i$ is.

## 3. Triangular fuzzy sets

In the following, we briefly introduce some basic definitions of triangular fuzzy sets. Definition 1 is the definition of triangular fuzzy number. Definition 2 shows the operational laws of triangular fuzzy numbers. Definition 3 gives the distance measure of triangular fuzzy numbers. These basic definitions will be used in next section, especially, the distance measure will be applied in Step 3 of Section 4.

**Definition 1.** (Fenton & Wang, 2006) A triangular fuzzy number $\tilde{a}$ is defined by a triple $(a_1, a_2, a_3)$. The membership function is defined as:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1), & \text{if } a_1 \leqslant x \leqslant a_2, \\ (a_3 - x)/(a_3 - a_2), & \text{if } a_2 \leqslant x \leqslant a_3, \\ 0, & \text{otherwise}. \end{cases} \tag{8}$$

The triangular fuzzy number is based on a three-value judgment: the minimum possible value $a_1$, the most possible value $a_2$ and the maximum possible value $a_3$.

**Definition 2.** (Yu & Hu, 2010) Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, where $a_1 \leqslant a_2 \leqslant a_3$ and $b_1 \leqslant b_2 \leqslant b_3$, the basic operations of triangular fuzzy numbers are defined as follows:

(1) Addition:
$\tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
(2) Multiplication:
$\tilde{a} \otimes \tilde{b} = (a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = (a_1 b_1, a_2 b_2, a_3 b_3)$.
(3) Division: $\tilde{a}/\tilde{b} = (a_1, a_2, a_3)/(b_1, b_2, b_3) = (a_1/b_1, a_2/b_2, a_3/b_3)$.

**Definition 3.** (Chen, 2000) Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, the distance between $\tilde{a}$ and $\tilde{b}$ can be calculated as:

$$\delta(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}. \tag{9}$$

## 4. An extended GRA method for MCDM with unknown weights

In fuzzy MCDM problems, performance rating values are usually characterized by fuzzy numbers. In this paper, criteria values are considered as linguistic variables. The concept of a linguistic variable is very useful in dealing with situations that are too complex or too ill-defined to be amenable for description in conventional quantitative expressions. These linguistic variables can be expressed as interval-valued triangular fuzzy numbers given in Table 1.