Fuzzy preference relations: Aggregation and weight determination

Ying-Ming Wang a,*, Zhi-Ping Fan b

a Institute of Soft Science, Fuzhou University, Fuzhou 350002, PR China
b School of Business Administration, Northeastern University, Shenyang 110004, PR China

Received 14 January 2007; received in revised form 4 May 2007; accepted 5 May 2007
Available online 13 May 2007

Abstract

Priority ranking and aggregation are two major concerns of fuzzy preference relations. This paper focuses on the aggregation of fuzzy preference relations and presents two optimization aggregation approaches to determine the relative weights of individual fuzzy preference relations so that they can be aggregated into a collective fuzzy preference relation in an additively optimal manner. Multiplicative preference relations can also be incorporated into the two optimization aggregation approaches, but need to be transformed into fuzzy format using appropriate transformation techniques. The proposed two approaches are tested and examined with two numerical examples and prove to be simple, effective and practical.

© 2007 Elsevier Ltd. All rights reserved.

Keywords: Fuzzy preference relation; Aggregation of fuzzy preference relations; Fuzzy group decision analysis; Priority ranking

1. Introduction

Increasing attention has been paid to fuzzy preference relations in the literature and more and more decision analyses use fuzzy preference relations to help decision makers (DMs) make their decisions. Priority ranking and aggregation are thought to be the two major concerns of the use of fuzzy preference relations. Quite a number of approaches have been developed to derive the priorities from a fuzzy preference relation. For example, Fodor and Roubens (1994) provided a net flow score procedure, which is similar to the extensively used multiple attribute decision making approaches such as PROMETHEE and ELECTRE (Brans, Vincke, & Mareschal, 1986; Brans & Vincke, 1985; Brans, Mareschal, & Vincke, 1984; Roy, 1991). Chiclana, Herrera, and Herrera-Viedma (1998, 2001) utilized fuzzy linguistic quantifiers and OWA operator to choose the best alternative from a collective fuzzy preference relation. Fernandez and Leyva (2004) proposed a multiobjective optimization method for deriving a ranking from a fuzzy preference relation. Xu and Da (2005) suggested a least deviation method to obtain the priority vector of a fuzzy preference relation. Xu (2004) also developed a

* Corresponding author. Tel.: +86 591 87893307; fax: +86 591 87892545.
E-mail address: msymwang@hotmail.com (Y.-M. Wang).

0360-8352/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cie.2007.05.001

As for the aggregation of fuzzy preference relations, the aggregation process may be linear or nonlinear. The most widely used aggregation approaches are fuzzy majority and OWA operator (Chiclana et al., 1998; Chiclana, Herrera, Herrera-Viedma, & Martínez, 2003; Fedrizzi, Kacprzyk, & Nurmi, 1993; Kacprzyk, Fedrizzi, & Nurmi, 1992). If preference information is given in the form of multiplicative rather than fuzzy preference relations, the ordered weighted geometric (OWG) operator guided by fuzzy majority are suggested (Chiclana et al., 2001; Herrera, Herrera-Viedma, & Chiclana, 2001). The use of these methods to aggregate individual fuzzy preference relations needs to define the membership functions of linguistic quantifiers representing fuzzy majorities and to choose their parameters very carefully. If the parameters are not chosen with much care, the collective fuzzy preference relation may not be additive reciprocal (Chiclana et al., 2003). Due to the fact that the membership functions are determined subjectively, the aggregation process cannot be optimal.

In this paper, we focus on the aggregation of fuzzy preference relations and propose two optimization aggregation approaches to assess the relative weights of individual fuzzy preference relations so that they can be aggregated into a collective fuzzy preference relation in an additively optimal manner. Multiplicative preference relations can also be incorporated into the optimization aggregation approaches, but need to be transformed into fuzzy format using appropriate transformation techniques.

The paper is organized as follows. Section 2 briefly reviews the net flow score procedure, which serves in this paper as the ranking approach for fuzzy preference relations. Section 3 proposes two optimization aggregation approaches to assess the relative weights of individual fuzzy preference relations so that they can be aggregated into a collective fuzzy preference relation in an additively optimal manner. Section 4 investigates two numerical examples using the proposed aggregation approaches to show their applications. The paper is concluded in Section 5.

### 2. The net flow score procedure for priority ranking

Let \( A = \{A_1, \ldots, A_n\} \) be a finite set of alternatives and \( E = \{E_1, \ldots, E_m\} \) the set of individuals, experts or decision makers (DMs). A fuzzy preference relation \( R \) is a fuzzy subset of \( A \times A \) characterized by the following membership function (Kacprzyk, 1986; Kacprzyk et al., 1992; Marimin, Hatono, & Tamura, 1998):

\[
\mu_R(A_i, A_j) = \begin{cases} 
1, & \text{if } A_i \text{ is definitely preferred to } A_j, \\
c \in (0, 1), & \text{if } A_i \text{ is slightly preferred to } A_j, \\
0.5, & \text{if there is no preference (i.e., indifference),} \\
d \in (0, 1), & \text{if } A_j \text{ is slightly preferred to } A_i, \\
0, & \text{if } A_j \text{ is definitely preferred to } A_i.
\end{cases}
\]  

(1)

\( R \) is a matrix of fuzzy preference relation, whose elements \( r_{ij} \) are given by \( r_{ij} = \mu_R(A_i, A_j) \), where \( r_{ij} + r_{ji} = 1 \) and \( r_{ji} = 0.5 \) (it could also be represented as \( r_{ij}^{(k)} = \ast - r \)) for all \( i, j = 1, \ldots, n \) and \( k = 1, \ldots, m \).

Since \( r_{ij} \) can be understood as the degree of preference of \( A_i \) over \( A_j \), \( \sum_{j=1, j \neq i}^n r_{ij} \) can accordingly be viewed as the total degree of preference of \( A_i \) over all the other \((n - 1)\) alternatives \( A_j \) (\( j = 1, \ldots, n; j \neq i \)), which can be referred to as leaving flow in the terminology of decision making and is denoted by

\[
\phi^+(A_i) = \sum_{j=1, j \neq i}^n r_{ij}, \quad i = 1, \ldots, n.
\]  

(2)

As such, \( \sum_{j=1, j \neq i}^n r_{ji} \) can be regarded as the total degree of preference of the \((n - 1)\) alternatives \( A_j \) (\( j = 1, \ldots, n; j \neq i \)) over \( A_i \), which can be called entering flow in the terminology of decision making and is denoted by