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Simple algorithms for updating multi-resource allocations in an unreliable flow network

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Abstract

In this paper, algorithms are developed to update the reliability-maximizing resource allocation in an unreliable flow network when either resource demand or the characteristic of the flow network changes. An unreliable flow network consists of nodes, characterized as source nodes, which supply resources of various types, sink nodes, at which resource demand takes place, and intermediate nodes, as well as unreliable directed arcs, which join pairs of nodes and whose capacities have multiple operational states. The network reliability of such an unreliable flow network is the probability that resources can be transmitted successfully from source nodes to sink nodes. With earlier developments on evaluating network reliability and resolving reliability-maximizing resource allocation problems in an unreliable flow network, one may recompute a new resource allocation strategy, when either resource demand or the characteristic of the flow network changes, without incorporating the efforts that have been made in obtaining the existing resource allocation. This study, in contrast, proposes updating, rather than recomputing, alternatives that take advantage of existing minimal path vectors and corresponding flow patterns. Procedural comparisons and numerical examples indicate that the updating schemes would perform better than the recomputing scheme in a large sized flow network that transmits various resource types.

Keywords: Flow network; Minimal path vector; Network reliability; Resource allocation

1. Introduction

This paper examines a resource allocation problem with varying demand in an unreliable flow network, which finds applications in real-world systems such as communication systems (Aggarwal, Chopra, & Bajwa, 1982; Samad, 1987), distributed computing systems (Ke & Wang, 1997), and electric and power systems (Clancy, Gross, & Wu, 1983). Such an unreliable flow network consists of a set of nodes, including resource-supplying nodes (*source* nodes), resource-demanding nodes (*sink* nodes), and intermediate nodes, as well as a collection of directed arcs joining pairs of nodes; its unreliability stems from operational uncertainties of arc capacities. Essential to reliable resource transmission in an unreliable flow network are assessment of network reliability and that of reliable resource allocation strategies.

The methodologies of evaluating network reliability in a single-source single-sink flow network have been extensively studied (Aggarwal et al., 1982; Aven, 1985; Doulliez & Jamoulle, 1972; Jane, Lin, & Yuan, 1993; Lin,

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Jane, & Yuan, 1995; Lee, 1980; Lin, 2001; Xue, 1985; Yeh, 2001a,b). In Lee (1980), for instance, the network reliability of a single-source single-sink flow network is defined as the probability that a specified flow demand can be transmitted through the flow network, and an evaluation method is developed that searches for all system states which satisfy the demand, using the notion of lexicographic ordering. In Barlow and Wu (1978), several properties of a multistate coherent system are generalized from those of a binary coherent system and a method for computing the probability that the maximal flow of the network meets a given system state is implied. The concepts of minimal path vectors to a given level and minimal cut vectors to a given level[1] are taken into account in Aven (1985); assuming that the minimal path vectors to a given level and the maximal cut vectors to a given level are known ex ante, two algorithms incorporating the state decomposition method (Doulliez & Jamoulle, 1972) are developed, respectively, to evaluate the network reliability of a multistate system. By relaxing the assumption that minimal path and maximal cut vectors are available, Xue (1985) applies discrete function theory to acquire all minimal path and maximal cut vectors in a multistate system prior to the evaluation of network reliability. The acquisition process of minimal path vectors and that of maximal cut vectors are further enhanced in Lin et al. (1995), Jane et al. (1993) and Yeh (2001b), respectively. In the case of unreliable nodes, algorithms for obtaining minimal path vectors are developed in Lin (2001) and Yeh (2001a).

The aforementioned studies focus on the development of algorithms for evaluating network reliability in a single-source single-sink flow network. In a multi-source multi-sink flow network, a resource allocation arises that is to allocate various resources at source nodes, subject to resource demand at sink nodes, in a reliable means so that resources can be transmitted successfully through the flow network. The studies (Hsieh & Chen, 2005; Hsieh & Lin, 2003) integrate resource allocation and reliability evaluation as a whole in an unreliable multi-source multi-sink flow network, and develop optimal resource allocation strategies for given resource demand.

In this study, we examine evaluation strategies for optimal resource allocations in unreliable multi-source multi-sink flow networks in which resource demand at sink nodes changes. Knowing that the flow patterns, which represent the resource quantities delivered from source nodes to sink nodes through various paths, for two similar demand configurations are similar to each other, we intend to update, rather than recompute, the resource allocation when current resource demand is slightly changed; and we focus in this paper on unitary changes in resource demand. We acknowledge that updating may not be appealing or feasible when current resource demand faces a dramatic change, resulting in very different flow patterns. Yet, if a variety of demand configurations are likely to take place, we could deploy the updating scheme to obtain optimal resource allocations incrementally for all demand configurations. We also examine the conditions under which updating is applicable when resource supplies at source nodes, the characteristics of arc capacities, and the maximum capacities of arcs change.

The remainder of this paper is organized as follows. Section 2 describes the model of an unreliable multi-source multi-sink flow network, and the reliability evaluation scheme and the procedure of computing the optimal resource allocation in such a flow network. Section 3 discusses the updating scheme when resource demand encounters a unit increase or a unit decrease, and develops two procedures for updating optimal resource allocations under unitary changes in resource demand. Section 4 discusses the cases for which the updating scheme will be beneficial under other changes such as changes in resource supply at some source node, the characteristic of some arc capacity, and the maximum capacity of some arc. Finally, Section 5 concludes with a brief summary.

2. Model of an unreliable flow network

We consider m types of resources to be transmitted through an unreliable multi-source multi-sink flow network which is represented by a graph $\mathbf{g}=(\mathbf{n},\mathbf{a},\mathbf{u})$, where $\mathbf{n}$ is the set of nodes and $\mathbf{a}=\{a_l|1\le l\le n\}$ is the set of directed arcs, joining pairs of nodes in $\mathbf{n}$. We assume that each arc $a_l\in\mathbf{a}$ is unreliable and has a maximum capacity $u_l\in\mathbf{u}$, and the capacity of a_l is a random variable taking on an integral value in the interval $[0,u_l]$. We also assume that all of the nodes in $\mathbf{a}$ are perfect and that during resource transmission no resource flow would disappear or be created in arcs and at nodes, except at source and sink nodes. We let $\mathbf{s}=\{s_1,\ldots,s_\sigma\}\subset\mathbf{n}$ and $\mathbf{t}=\{t_1,\ldots,t_\theta\}\subset\mathbf{n}$ denote the source nodes at which resources are supplied to the flow network and the sink nodes at which resources are demanded, respectively.

[1] Minimal path and maximal cut vectors are referred to, respectively, as lower and upper boundary points in Hudson and Kapur (1983, 1985), upper and lower critical connection vectors in El-Neweihi, Proschan, and Sethuraman (1978), minimal upper and maximal lower vectors in Xue (1985), and d-MPs and d-MCs in Jane et al. (1993).