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Reliability sampling plans for Weibull distribution with limited capacity of test facility ☆

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Abstract

This paper establishes reliability sampling plans for the Weibull lifetime distribution based on type II censored data with limited capacity of test facility. The products are sold under a general rebate warranty policy. It is also assumed that the shape parameter of the lifetime distribution is known, and the scale parameter is a random variable varying from lot to lot. A cost model is established which contains the cost per unit on test, the cost per unit time for life test, and the costs of rejecting and accepting a unit. An algorithm for determining the optimal reliability sampling plans which minimize the expected average cost per lot is provided. Some numerical results to illustrate the use of the proposed method are studied.

Keywords: Acceptance sampling; Posterior distribution; Prior distribution; Type II censoring; Warranty policy

1. Introduction

Acceptance sampling technique is often used to determine whether a lot of products is accepted or not based on only inspecting the quality of a small set of products. In many practical applications, an important quality variable is the lifetime of a product. Acceptance sampling plans used to determine the acceptability of a lot of products with respect to their lifetimes are called reliability sampling plans.

In most life testing plans, generally there are constraints on the length of life tests and, as a result, data have to be analyzed before all units have failed. Censoring arises in a life test when exact lifetimes are known for only a portion of test units and the remainder of the lifetimes are known only to exceed certain values under an experiment. There are several types of censored test. One of the most common censoring schemes is type II censoring. In a type II censoring, a total of n units is placed on test, but instead of continuing until all n units have failed, the test is terminated at the time of the r-th ($1 \leqslant r \leqslant n$) unit failure. The designs of reliability sampling plans under different censoring schemes have been studied by many researchers. Fertig and Mann (1980)

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provided a life test sampling plan for two-parameter Weibull distribution. Schneider (1989) discussed the sampling plans with type II censoring for the Lognormal and Weibull distributions. Balasooriya (1995) developed failure-censored reliability sampling plans for the two-parameter exponential distribution. Wu and Tsai (2000) proposed a failure-censored sampling plan for the Weibull distribution. Wu and Tsai (2005) and Tsai and Wu (2006) developed truncated life test plans for Birnbaum–Saunders and generalized Rayleigh distributions, respectively.

The Bayesian methods arise naturally when prior information is available for planning and estimation. Combinations of extensive past experience and engineering knowledge can provide prior information to form a framework for inference and decision making. Sampling plan with relevant prior information can reduce needed experimental resources. In the literature, Fertig and Mann (1974) investigated Bayesian reliability sampling plans under a linear loss function. Nigm and Ismail (1985) considered Bayesian sampling plans for the two-parameter exponential distribution. Lam (1988) and Lam and Lau (1993) developed some optimal Bayesian sampling plans with polynomial loss function. Zhang and Meeker (2005) described Bayesian methods for life test planning with type II censored data from a Weibull distribution when the shape parameter is given.

One practical problem arising from designing a reliability sampling plan is the cost of experiment. The cost model consists of the cost per unit on test, the cost per unit time for life test, and the costs of accepting and rejecting a unit. Nowadays, products are usually sold to the consumer with a warranty policy. Hence, it is important to consider the costs associated with the warranty policies. Thomas (1983), Nguyen and Murthy (1984), Kwon (1996) discussed some warranty policies such as failure free policy, prorated rebate policy and general rebate policy.

In many practical situations, the capacity of test facility is limited. It may not be possible to place all n units on test simultaneously. Hence, the optimal Bayesian reliability sampling plans obtained in the literature may not be adequate. In this paper, we investigate the reliability sampling plan based on Bayesian method for products with the Weibull lifetime distribution under general rebate warranty policy and limited capacity of test facility. The rest of this paper is organized as follows. In Section 2, the proposed sampling plan is established. In Section 3, some numerical studies are provided to illustrate the proposed method. Results of sensitivity analysis are presented in Section 4 and conclusions are made in Section 5.

2. The sampling plan

Suppose that the lifetimes of the products being tested have a Weibull distribution with probability density function (p.d.f.) and cumulative density function (c.d.f.), respectively,

$$f(t;\theta,\beta) = \frac{\beta}{\theta} t^{\beta-1} e^{-\frac{t^\beta}{\theta}}, \quad t > 0 \tag{1}$$

and

$$F(t;\theta,\beta) = 1 - e^{-\frac{t^\beta}{\theta}}, \quad t > 0,$$

where $\beta > 0$ is the shape parameter and $\theta > 0$ is the scale parameter. In this paper, we assume that the value of β is known. Soland (1968) gave some discussions for this assumption. We also assume that the scale parameter θ is a random variable varying from lot to lot according to a specified distribution. We consider the conjugate prior of the form

$$g(\theta) = \frac{b^a}{\Gamma(a)} \theta^{-(a+1)} e^{-\frac{b}{\theta}}, \quad \theta > 0, \tag{2}$$

where $a > 0$ and $b > 0$ are known constants and $\Gamma(\cdot)$ denotes the gamma function. This density is known as the inverted gamma distribution. The choice of inverted gamma prior for θ is equivalent to selecting a gamma prior for $\lambda = \frac{1}{\theta}$. The reader can follow the method proposed by Waller, Johnson, Waterman, and Martz (1977) to choose the parameters of the gamma prior distribution.

Suppose that the lots of size N are submitted for inspection and the life testing with type II censoring is considered. In a type II censoring scheme, a total of n units drawn at random from a lot are put on test simultaneously, but instead of continuing until all n units have failed, the life test is stopped at the time of the r-th