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A mathematical model for the management of a Service Center

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ABSTRACT

In this paper we propose a mathematical model to manage a Service Center (*SC*) which is based on a system of ordinary differential equations. By resorting to this model, the manager of the *SC* can design planning strategies to satisfy customer orders, under strict deadlines and human resource constraints. After describing the model, we introduce criteria which optimize the processing time and supply a more accurate description of working behavior. Finally, we conclude presenting some numerical simulations which demonstrate the usefulness of the proposed model to reach correct decisions in managing a *SC*.

1. Introduction

This paper originates from a research project[1] funded both to formulate a mathematical model and to implement a software for the management of a Service Center (*SC*). The problem we faced can be sketched as follows: the manager of a *SC* must decide which customer orders with *strict deadlines* can be chosen in a set of potential orders; then, he must determine a *schedule* for all the selected orders in such a way that all the orders are dealt with before their due dates. The set of orders (*jobs*) is a priori partitioned into a set of *job families* $\{\mathrm{JF}_1, \mathrm{JF}_2, \ldots, \mathrm{JF}_m\}$ so that all the jobs of each family has the same deadline, although the arrival time of any job could be different.[2] For processing the jobs in the due times, the manager can resort to a set of *heterogeneous human resources*, to which he could decide to add part time workers or hired extra resource units. This problem belongs to the *dynamic framework* in which inevitable and unpredictable *real-time events* may cause a change in the schedule plans. For instance, these events could refer to the *resources* (i.e. ill operators, unavailability or tool failures, etc.), and to the *jobs* (i.e. cancellation of an order, early or delayed arrival of jobs, a change in job priority, a change in job processing time, etc.).

The model we propose has the purpose of guiding the manager of the *SC* to choose among the potential orders in such a way as to answer the following questions:

- Let us suppose that a given number of orders have already been accepted by the *SC*. When the different deadlines and the workload are taken into account, is it possible to determine the date in which they will be completed?
- Is it possible to know if a further order can be accepted when the workload is considered?
- If the processing times are not compatible with the deadlines of the different orders, how must the schedule plans be modified?

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2 Since in our case, the time list of arrivals is unknown or given with a probability distribution, we decided to call this list *arrival time list* instead of *release time list*.

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- If the ordinary conditions, which allow the correct behavior of the already accepted orders in the due times, are modified by unexpected events (for instance the unavailability of resources due to ill operators or shifts of operators to other jobs), what is the resulting delay? How could it be reset?
- For a given set of activities to be developed and a workforce formed by different skills, which workers or resources should be assigned to the different activities in order to meet all the deadlines?
- If the scarcity of the resources produces bottlenecks and consequent delays in the production cycle, is it possible to adopt suitable strategies to meet the deadlines?

The above considerations suggest that the problem can naturally be formulated as a *scheduling problem*. In fact, we must find for each job the *best possible schedule* such that all the above *constraints* are met and a suitable given *objective function* is optimized (i.e. the total costs are minimized, the maximal lateness of the jobs are minimized, the total net profit is maximized, etc.). Due to the rich variety of different problems within this research field, scheduling is one of the most widely inter-disciplinary research areas (Mathematics, Operations Research, Information Technology and Computer Science, Manufacturing, Management, Business, Engineering, etc.). That is proved by more than 30,000 publications, referring to "scheduling" since the 1990s, that can be found in *Web of Science*. This situation makes an exhaustive state-of-the-art review on the scheduling problem very difficult. For instance, the recent review papers are addressed to a determined class of scheduling problems (see, for instance, [1] on scheduling problems arising in production industries, [2] on scheduling problems with setup considerations, [3] on dynamic scheduling in manufacturing systems, [4] on multicriteria scheduling, [5] on scheduling with learning effects, and so on). We remark that our problem can be classified as a *workforce scheduling problem*, for which the numerous approaches existing in literature use many different objective functions and constraints, as well as a wide variety of mathematical models (see [6–13] and reference therein). Moreover, it is well known that almost all the scheduling problems are typically *NP-hard*, i.e., so far there does not exist any reliable numerical method able to find an *optimal solution* within a reasonable time. This difficulty pushed researchers to develop heuristic algorithms or approximation procedures (see [1,2,14,15]).

Since the manager's decision of accepting or rejecting an order must be taken in *real-time*, we were asked to propose a mathematical model to be written in finite arithmetic by using an algorithm compatible with the database and the tool of the Work Flow Management (*WFM*), already employed by *SC*.[3]

In order to solve our problem in real-time, we propose an *evolutive mathematical model* which evaluates a *feasible not optimal scheduling* for the job families. In particular, it ensures that all the deadlines are met, provided that the potential customer orders with their characteristics (such as process route, processing times, deadline, etc.), and the job shop configuration (such as resources, their availability, etc.) are assigned. This model is based on a system of ordinary differential equations (*ODEs*) which, in the first approximation, can analytically be solved. Further, for these equations a numerical procedure is implemented to answer in real-time all the above managerial questions. We conclude by noting that resorting to a mathematical model of a *feasible not optimal scheduling* does not always represent a real simplification of the problem. For instance, in the case in which to meet the deadline of a particular order requires an increasing of the number of the people involved in the work, it would appear quite spontaneous the choice leading to a positive balance between costs and profits. However, the manager could decide to accept this new order as a business strategy (important customer, prestigious order, etc.), even if there is not a total net profit. In these situations, the choice to adopt is devolved upon the manager since it could be not convenient in pursuing a prefixed objective.

The paper is organized into five sections including the introduction. In Section 2 we describe the proposed mathematical model, whereas in Section 3 we present suitable criteria to resect the dead times. Section 4 is devoted to some numerical simulations relative to the model which is based on the first choice of the involvement coefficients $a_h^{i,j}$ defined in Section 3.1. Finally, Section 5 deals with the research perspectives concerning suitable development of the model in a classical framework of combinatorial optimization problems.

2. A mathematical model

Before discussing the mathematical model, we describe the fundamental quantities characterizing both the structure of *SC* and the activities of any job.

A set of job families $\{\mathrm{JF}_1, \mathrm{JF}_2, \ldots, \mathrm{JF}_m\}$ collect all types of possible orders, and with each job $j_h^i \in \mathrm{JF}_i$, for $i = 1, \ldots, m$ and $h = 1, \ldots, |\mathrm{JF}_i|$, a deadline d_i and an arrival time t_h^i are associated. In our problem each job belonging to the family JF_i consists of a set of n_i *sequential activities* $\left\{A_{i,1}, A_{i,2}, \ldots, A_{i,n_i}\right\}$:

$$\text{Job family } \mathrm{JF}_i \equiv \begin{cases} \text{Activity } A_{i,1} \\ \text{Activity } A_{i,2} \\ \ldots \\ \text{Activity } A_{i,n_i-1} \\ \text{Activity } A_{i,n_i}. \end{cases}$$

[3] In other words, the SC wished that the implementation of our model became an upgrade of the software Mentore 5.0 already used by the *SC*.