Available online at www.sciencedirect.com
ScienceDirect
Mathematical and Computer Modelling 47 (2008) 894–902
MATHEMATICAL AND COMPUTER MODELLING
www.elsevier.com/locate/mcm

# A simple method to synchronize chaotic systems and its application to secure communications

Jiin-Po Yeh*, Kun-Lin Wu

*Department of Civil and Ecological Engineering, I-Shou University, Kaohsiung County 840, Taiwan*

Received 1 May 2006; received in revised form 8 June 2007; accepted 22 June 2007

**Abstract**

This paper proposes a simple method to synchronize two Lorenz systems with different parameters. One of the Lorenz systems is driven by a variable in the other system. The time series of the driving variable in the drive system and its counterpart in the response system are collected. With the parameters in the response system updated based on skills introduced in this paper, the correlation coefficient and the standard deviation ratio between the time series of the drive and the response system will gradually approach one, which, found by this paper, will cause these two systems to be synchronized. The best set of parameters found for the response system is the one best satisfying the above two conditions. This paper also extends the above idea to secure communications, where the previous driving variable is used to mask an information signal. Using the same strategy as synchronization, these two systems can be synchronized and consequently, the information signal recovered with sufficient precision, as long as the frequency of the information signal is outside the range of the main frequencies of the masking variable and its power level is lower than those of the main frequencies.
© 2007 Elsevier Ltd. All rights reserved.

*Keywords:* Synchronize; Correlation coefficient; Standard deviation; Secure communications

## 1. Introduction

In recent years, there have been a large number of publications devoted to research on chaotic signals based only on the input–output data due to lack of the mathematical model. An artificial neural network with one input layer and some hidden layers is often seen to identify or forecast a chaotic system, after the weights associated with input-to-hidden connections and the weights associated with hidden-to-hidden and hidden-to-output connections are decided [1–4]. A fuzzy model composed of a number of membership functions associated with their respective parameters can also be used to identify, predict or control chaotic systems [5–7]. Furthermore, studies on the synchronization of two chaotic systems have also been intensive over the past decade or so, such as the general concept of synchronization in chaotic systems [8], the issues of synchronization from the perspective of Generalized Hamiltonian systems including dissipation and destabilizing terms where the possibilities of several chaotic systems for synchronization were confirmed [9], synchronization of two identical chaotic systems with different initial values

* Corresponding author.
*E-mail address:* jpyeh@isu.edu.tw (J.-P. Yeh).

0895-7177/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2007.06.021

by carefully designing an invariant manifold with an appropriate controller attached to one equation of the response system. [10], synchronization of two Lorenz systems with different initial values by introducing one control function to each equation of the response system [11–13], and a hybrid (generalized plus identical) chaos synchronization using Rossler chaotic oscillator as the platform [14], as well as the demonstration of secure communication via the synchronization of two chaotic systems with the same components, where an information signal is injected into a chaotic signal, transmitted, and then detected and recovered by the receiver [15–18].

This paper also explores the topics of synchronization and secure communication, but uses techniques based only on the knowledge of statistics, which is a distinction from the above-mentioned papers. To make the correlation coefficient between the time series of one drive variable in the drive (or transmitting) system and that of its counterpart variable in the response (or receiving) system as well as the ratio of standard deviation between these two series alternately close to one, the parameters in the response (or receiving) system are gradually modified until the synchronization of these two chaotic systems is achieved with sufficient precision. Once the synchronization between these two chaotic systems is achieved, the parameters in the driving (or transmitting) system are revealed and the recovery of the information signal can also be assured. Details will be provided throughout the paper. This paper is arranged as follows. Section 2 shows both the drive and response chaotic systems. Section 3 illustrates the proposed method. Numerical results are presented to demonstrate the effectiveness of the proposed method in Section 4. Section 5 is devoted to some conclusions.

## 2. Construction of the drive–response pair

The dynamical system used in this paper is the Lorenz system [19], one of the first models shown to exhibit chaotic behavior in numerical simulation. It is a fluid convection model with three state variables $(x, y, z)$. The variable $x(t)$ represents a measure of the fluid velocity, while $y(t)$ and $z(t)$ represent measures of the spatial temperature distribution in the fluid layer under gravity. This mathematical model of convection was first derived by Saltzman [20] and simplified by Lorenz. The nondimensional forms of Lorenz's equations are

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= -x_1x_3 + bx_1 - x_2 \\ \dot{x}_3 &= x_1x_2 - cx_3. \end{aligned} \tag{1}$$

The parameters $a$ and $b$ are related to the Prandtl number and Rayleigh number, respectively, and the third parameter $c$ is a geometric factor. The three parameters $a, b$ and $c$ are chosen to be in the chaotic regime: $a = 10$, $b = 28$, and $c = 2.67$. The first signal $x_1(t)$ of system (1) is chosen to drive another Lorenz system

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) \\ \dot{y}_2 &= -x_1y_3 + \beta x_1 - y_2 \\ \dot{y}_3 &= x_1y_2 - \gamma y_3 \end{aligned} \tag{2}$$

where the parameters $\alpha$, $\beta$ and $\gamma$ are randomly selected in the beginning. Eq. (1) and Eq. (2) are called the drive and response systems, respectively. The behavior of the subsystem $(y_2, y_3)$ is dependent on the signal $x_1$, but the behavior of $x_1$ is not influenced by the behavior of $y_2$ and $y_3$. The reason to choose the subsystem $(y_2, y_3)$ driven by the signal $x_1$ is because its conditional Lyapunov exponents are negative [21,22]. The negativity of the conditional Lyapunov exponents is a necessary condition for the stability of the subsystem $(x_2, x_3)$ or $(y_2, y_3)$.

## 3. The proposed method

Synchronization means the trajectories of one of the systems will converge to the same values as the other and they will remain in steps with each other. To reach synchronization, this paper presents an easy-to-implement method. Define the state errors between the drive system and response system as $e_1 = x_1 - y_1$, $e_2 = x_2 - y_2$ and $e_3 = x_3 - y_3$. Subtracting the first differential equation of system (2) from that of system (1) leads to

$$\dot{x}_1 - \dot{y}_1 = \dot{e}_1 = (ax_2 - \alpha y_2) - (ax_1 - \alpha y_1). \tag{3}$$

Download English Version:

https://daneshyari.com/en/article/1137582

Download Persian Version:

https://daneshyari.com/article/1137582

Daneshyari.com