Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 129 (2016) 81–93
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

An algebraic approach for detecting nearly dangerous situations in expert systems

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Received 6 October 2012; received in revised form 14 February 2016; accepted 6 April 2016
Available online 16 April 2016

Abstract

The aim of this work is to present theoretically a new algebraic method for detecting nearly dangerous states in an expert system whose knowledge is represented by propositional Boolean logic. Given a dangerous state which does not happen at present, our method is able to detect if a dangerous situation would happen if an input variable of the Expert System changed. The method presented here is based on calculating just one Groebner basis of a polynomial ideal representing the system knowledge. In this way, although the Expert System is designed to notify only dangerous situations, we have developed an algebraic model for including in the expert system the ability to recommend watching carefully some Boolean variables such that if one of them changed, the system would fall in a dangerous situation. In this way, the same expert system not only warns about dangerous situations but also about nearly dangerous situations. It may be noted that our method does not require to change the Expert System for including this facility. As far as our knowledge goes, our work is completely new in the field of Expert Systems.

Keywords: Rule based expert systems; Logic and symbolic computing; Groebner bases

1. Introduction

Rule based expert systems (RBES) are computational programs on a certain domain which try to simulate the decisions that human experts on this domain would take. An interesting way for representing knowledge in an expert system is based on describing it through propositional logic[1]. By means of a mathematical result [7] based on previous work [1,4,9,10,17,6], that issue can be transformed to an algebraic problem. In this way, RBES based on propositional logic may be very easily implemented by means of a computer algebra system like CoCoA [3] or Polybori [2]. Making use of this result, different expert systems have been so far developed in recent years [11,12,14,13,18,8,15].

RBES may be specially interesting in the domain of the detection of dangerous situations [8,15]. In this domain, a RBES tries to automatically infer whether the situation (specified through input Boolean variables) may be regarded

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[1] We have also developed other algebraic models when using knowledge representation models different from propositional logic [5].

http://dx.doi.org/10.1016/j.matcom.2016.04.001

as dangerous or not. Naturally, according to the result presented in [17,7] this kind of expert systems may be easily implemented using computer algebra systems.

In this paper we are concerned with providing this kind of RBES with the facility of warning not only about dangerous situations but also about nearly dangerous situations, namely situations which are not dangerous yet, but would become so if an input variable would suddenly change its value. In [16], we also focus on expert systems detecting dangerous situations. In this previous paper, we considered situations on which the input is not completely known (since we ignore the value of some of these input Boolean variables) and which could result to be dangerous if these input variables took an appropriate value. Our aim here is different: we assume that all the Boolean input variables are completely known, and our goal is now to detect if a situation could become dangerous if one of these variable changed its value.

A traditional way for providing RBES with this facility would involve to change the knowledge base of the expert system by adding new rules which were able to detect this nearly dangerous situation. In this paper, we will present a new algebraic approach which allows to provide the RBES with this facility without changing the knowledge base of the RBES. As far as we are concerned, our work is completely new in the field of RBES.

In Section 2 we will review some particular aspects of RBES and computer algebra required for this paper. In Section 3, we will formally define the concept of nearly dangerous situation. In Section 4, we will describe our algebraic approach for finding nearly dangerous situations. In Section 5, we show an example of how our technique works. Finally, in Section 6 we discuss the conclusions and possible extensions of our technique.

2. Some introductory notes about RBES and Groebner bases

2.1. Notes about RBES based on Boolean propositional logic

In this section, we will describe some outlines of a RBES whose knowledge is represented by formulae in Boolean propositional logic.

In Boolean propositional logic, given a finite set of Boolean variables $X_1, \ldots, X_m$, we may build formulae using the Boolean connectives $\neg$, $\vee$, $\wedge$, and $\rightarrow$ (although the connective $\rightarrow$ may be derived from the other connectives, we have included it due to its importance in RBES).

We will make use of $\mathcal{C}$ to denote the set of formulae.

By its importance, we distinguish formulae with the form:

$$(A_1 \wedge A_2 \wedge \cdots \wedge A_k) \longrightarrow (A_{k+1} \vee \cdots \vee A_n)$$

where $n \geq 2$, and each formula $A_1, \ldots, A_n$ is either a Boolean variable (X) or the negation of a Boolean variable $(\neg X)$. This kind of formulae are termed as *rules*. As may be seen, any formula with at least a connective $\vee$, $\wedge$ or $\rightarrow$ in Boolean propositional logic may be described (rewritten) as a rule.

On this ground, we may consider that a RBES based on Boolean propositional logic is composed by these three elements:

Input The input of a RBES is concerned with the information related to the environment of the RBES. This input is described by means of a set of formulae, $\mathcal{F} \subset \mathcal{C}$, with the form $\neg X$ or X where X is a Boolean variable. The Boolean variables used to describe the input are termed as input variables.

Output The output of a RBES is concerned with the information inferred by the RBES which will be useful for performing actions in the environment. Here we will consider just one variable Z which informs if the situation is dangerous or not. That is to say, if the system infers the formula Z, we state that the situation is dangerous.

Knowledge-Base The knowledge-base of the RBES is concerned with the information contained in the system, which is used along with the input of the RBES to infer the output of the system. In a RBES based on propositional Boolean logic, this knowledge-base will be mostly represented by a finite set of rules, $\mathcal{K} \subset \mathcal{C}$. Obviously, this set of rules may require to define new auxiliary Boolean variables.

The question about how to formalize the concept of inference in a RBES based on propositional Boolean logic is related to the well-known concept of tautological consequence in propositional logic. As usual, we make use of