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Mathematical basis for a mixed inverse scattering problem

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Abstract

In this paper we consider the scattering of an electromagnetic time-harmonic plane wave by an infinite cylinder having an open arc and a bounded domain in R^2 as cross section. To this end, we solve a mixed scattering problem for the Helmholtz equation in R^2 where the scattering object is a combination of a crack Γ and a bounded obstacle D, and we set suitable boundary conditions on Γ and ∂D ($\partial D \in C^2$). The boundary integral method is employed to study the direct scattering problem. The mathematical basis is given to reconstruct the shape of the crack and the obstacle by using the linear sampling method. The numerical examples are given to show the viability of the method.

Keywords: The linear sampling method; Inverse scattering problem; Mixed scattering problem

1. Introduction

In 1995, Kress considered the direct and inverse scattering problem for a perfectly conducting crack, and used integral equation method to solve both the direct and inverse problems for a sound-soft crack. The scattering problem in the unbounded domain is thus converted into a boundary integral equation. Mönch [14] extended this approach to a Neumann crack in 1997. In 2000, Kress's work was continued by Kirsch and Ritter in [8] who used the factorization method to reconstruct the shape of the crack from the knowledge of the far-field pattern, and in the same year these results were generalized to the scattering problem with cracks for Maxwell equations in [1] by Ammari, Bao and Wood. Later in 2003, F. Cakoni and D. Colton in [2] discussed the direct and inverse scattering problems for cracks (possibly) coated on one side by a material with surface impedance λ. Extending to the impedance problem, Kuo-Ming Lee [12] considered the direct and inverse scattering problem for an impedance crack in 2008. In 2013, Hassen, Boukari and Haddar discussed a kind of scattering problem with impedance cracks in [7], and they reconstruct the shape of the crack by using the linear sampling method. More related results can be found in [2,3,5,6,9,15,16] and the reference therein.

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Motivation: If the scatterer is not a simple crack, for example, it is a combination of a crack and a bounded domain, how is the scattering problem? In this paper, we consider the problem of scattering of electromagnetic waves from an infinite cylinder (not perfectly) having an open arc Γ and a bounded domain D in R^2 as cross section. In this work we assume that the obstacle is thin dielectric cylinder whose properties do not change along the axis and the incident time harmonic electric field is polarized parallel to the cylinder axis. After factoring out the term $e^{-i\omega t}$, where ω is the fixed frequency, the only non zero component u of the total electric field satisfies the Helmholtz equation in the domain $R^2 \setminus (\bar{D} \cup \bar{\Gamma})$, that is

$$\Delta u + k^2 u = 0, \quad \text{in } R^2 \setminus (\bar{D} \cup \bar{\Gamma})$$

where k is the wavenumber, and we assume that $k > 0$ for simplicity.

For the scattering problem, we usually assume the scatterer is a bounded domain, or a simple crack (or several cracks), and there is a few results about the mixed scattering problem, that is, the scatterer is a combination of a bounded domain and a crack. The inverse problem, we usually consider is, given the wave number and the far field pattern, to determine the shape of the arc Γ, or the shape of D, or both Γ and D. As far as we understand, it is a difficult thing to reconstruct both Γ and D simultaneously. This is the motivation of this paper. The reason why we choose the boundary conditions on both Γ and ∂D (see the following section) is that the mathematical treatment is convenient.

As we know, the reality is often very complex. This model corresponds to applications in biomedical imaging, non-destructive testing, and geophysical explorations. For example, we want to look for some things under the sea (e.g. the wreckage of a plane) by using radar or sonar, the scatterer may be a combination of all kinds of obstacles such as a bounded fragment and a piece of cable.

Briefly speaking, in this paper we consider the scattering of an electromagnetic time-harmonic plane wave by an infinite cylinder having an open arc Γ and a bounded domain D ($\partial D \in C^2$) in R^2 as cross section. We assume that the crack is coated on one side by a material with surface impedance λ. This corresponds to the situation when the boundary or more generally a portion of the boundary is coated with an unknown material in order to avoid detection. Assuming that the electric field is polarized in the TM mode, this leads to a mixed boundary value problem for the Helmholtz equation defined in the exterior of an open arc and a bounded domain in R^2.

The goal of this paper is to give a mathematical basis to reconstruct the shape of the crack and the obstacle by using the linear sampling method. However, the well posedness of the direct scattering problem is necessary. So, in the first part of this paper, we establish the existence and uniqueness of a solution to this direct scattering problem by using the method of boundary integral equations (see [5,6] and the reference therein). The main challenge is to derive a suitable boundary integral system and show that the corresponding boundary integral operators are Fredholm of index zero. Then, the well-posedness of the solution to the direct problem can be obtained by employing the Fredholm theory. To the corresponding inverse scattering problem, the main challenge is to establish some properties to the far field operator F, such as compactness and injectivity which can be used to ensure the numerical implementation.

This paper is organized as follows. In Section 2, we will introduce the direct scattering problem, the well posedness of the direct scattering problem is given by using the boundary integral method. In Section 3, we state the mathematical basis to reconstruct the shape of the crack and the obstacle by using the linear sampling method, and some numerical examples are given to show the viability of the method in Section 4.

2. Formulation of the direct and inverse scattering problem

Consider the scattering of time-harmonic electromagnetic plane waves from an infinite cylinder having an open arc Γ (smooth) and a bounded domain D ($\partial D \in C^2$) in R^2 as cross section. For further considerations we extend the arc Γ to an arbitrary smooth, simply connected, closed curve ∂G ($\partial G \in C^2$) enclosing a bounded domain G such that the normal vector ν on Γ coincides with the normal vector on ∂G which we denote by ν. Notice that ν is the unit normal vector defined almost everywhere on ∂G and ∂D and direct into the exterior of G and D. We assume that the domain D is completely contained in G, i.e., $D \subset G$ and $\partial D \cap \partial G = \emptyset$. Throughout this paper, we shall assume that $\partial D \in C^2$ and $\partial G \in C^2$.