Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 119 (2016) 91–107
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

An impulsive two-stage predator–prey model with stage-structure and square root functional responses

Xiangmin Ma, Yuanfu Shao*, Zhen Wang, Mengzhuo Luo, Xianjia Fang, Zhixiang Ju

School of Science, Guilin University of Technology, Guilin, Guangxi 541004, PR China

Received 12 June 2015; received in revised form 14 July 2015; accepted 13 August 2015
Available online 21 August 2015

Abstract

Taking into account stage-structure for predator and periodic pulse at different fixed moment, a delayed predator–prey system with square root functional response is investigated in this paper. Sufficient conditions for the global attractivity of the mature predator-extinction periodic solution are obtained. Further, by using theories of impulsive differential equation and delay differential equation, we investigate the permanence of this system. Finally, examples and numerical simulations are given to show that time delays and pulses play an important role in the dynamics which makes the system be more complex.

Keywords: Stage structure; Impulsive control; Functional response; Stability; Permanence

1. Introduction

In recent management of renewable natural resources, biologists have been aware of the possibility that suitable stocking and harvesting can alter the genetic pattern of resources and play a key role in the permanence of ecosystems [12]. As a matter of fact, human harvesting or stocking always happens in a short time or instantaneously, hence the continuous action of human should be replaced. Recently, impulsive differential equations have been extensively used to the models in chemistry, engineering, biology, physics and other sciences, with particular emphasis on population dynamics [7,14,22–24], and more complex models are developed and applied to the field of agricultural management [10,16,17,19,20].

Actually, many species have the life history that goes through two stages: immature and mature, which have many behavioral differences between them. For example, an immature predator has no ability to capture its prey directly, and it needs time to get the ability. Therefore, it is more rational to model predator–prey system by stage-structured model, where the stage-structure is modeled by using a constant time delay. Aiello and Freedman [1] studied a basic stage-structured single-species model as follows,

$$\begin{cases} y_1'(t) = ky_2(t) - ke^{-d\tau} y_2(t-\tau) - dy_1(t), \\ y_2'(t) = ke^{-d\tau} y_2(t-\tau) - \eta y_2^2(t), \end{cases}$$

* Corresponding author.
E-mail address: shaoyuanfu@163.com (Y. Shao).

http://dx.doi.org/10.1016/j.matcom.2015.08.009
0378-4754/

where $y_1(t)$ and $y_2(t)$ represent the immature and mature population densities, respectively, at any time $t > 0$. Here they let τ be the maturation time delay, $ke^{-d\tau} y_2(t-\tau)$ represents the number of immature population who is born at time $t-\tau$ and survive at time t. In recent years, the dynamics of mathematical models with stage-structured prey–predator model has been extensively studied, which greatly enriched biological background [11,25,28].

In real world, mathematical population models take many forms. Depending on the time scale and space structure of concrete problem, it can be modeled by integral equations, different equations, ordinary differential equations, impulsive differential equations [18], partial differential equations, delay differential equations [29], or the combination of these forms [5,6,9], and these equations exhibit well known properties which are consistent with the real world. Jiao and Meng [13] proposed a delayed stage-structured predator–prey system with impulsive stocking prey, which is given by

$$\begin{cases} \left.\begin{aligned} x'(t) &= x(t)(a-bx(t)) - \frac{\beta x(t)}{1+\alpha x(t)} y_2(t), \\ y_1'(t) &= k\frac{\beta x(t)}{1+\alpha x(t)} y_2(t) - ke^{-w\tau}\frac{\beta x(t-\tau)}{1+\alpha x(t-\tau)} y_2(t-\tau) - wy_1(t), \\ y_2'(t) &= ke^{-d_1\tau}\frac{\beta x(t-\tau)}{1+\alpha x(t-\tau)} y_2(t-\tau) - d_3 y_2(t) - Ey_2(t) \end{aligned}\right\} \quad t \neq nT. \\ \left.\begin{aligned} x(t^+) &= x(t)+\mu, \\ y_1(t^+) &= y_1(t), \\ y_2(t^+) &= y_2(t), \end{aligned}\right\} \quad t = nT,\ n = 1, 2, \ldots. \end{cases} \tag{1.1}$$

In model (1.1), $x(t)$, $y_1(t)$ and $y_2(t)$ represent the densities of prey, immature predator and mature predator at time t, respectively. Parameter $a > 0$ is the intrinsic growth rate of prey, $b > 0$ is the coefficient of intraspecific competition, β is capture rate of mature predator, k is conversion rate of nutrients into the reproduction of the mature predator, w and d_3 are death rates of immature predator and mature predator, respectively. τ is a constant time delay to maturity, $0 < E < 1$ is the effect of continuous harvesting for predator, and μ is impulsive stocking amount of prey at time $t = nT$, $n = 1, 2, \ldots$. The predation term $\frac{\beta x(t)}{1+\alpha x(t)}$ is Holling type II functional response of the predator to prey. More details about this function see [4]. The authors assumed that immature individuals and mature individuals are divided by age τ, and immature individual predators do not feed on prey and do not have the ability to reproduce. By constructing suitable Lyapunov functional, they obtained sufficient conditions of the global attractivity of predator-extinction boundary periodic solution and the permanence of system (1.1), which showed that the behavior of impulsive stocking prey played an important role in the permanence of this system.

However, if examining the more complicated case that the prey exhibits herd behavior, i.e., the predator interacts with the prey along the outer corridor of the herd of prey, then the predation term in (1.1) cannot more accurately reflect the dynamic behavior of individuals. As Ajraldi and Pittavino [2] argued that, it was more appropriate to model the response functions of prey that exhibited herd behavior in terms of the square root of the prey population, which may be entirely fitting for herbivores on a large savanna and their large predators. Subsequently, Braza [4] proposed the following predator–prey model:

$$\begin{cases} x'(t) = x(t)(a-bx(t)) - \dfrac{\beta\sqrt{x(t)}}{1+\alpha\sqrt{x(t)}} y_2(t), \\ y'(t) = k\dfrac{\beta\sqrt{x(t)}}{1+\alpha\sqrt{x(t)}} y(t) - dy(t). \end{cases} \tag{1.2}$$

By simplifying assumption, the author studied the case of $\alpha = 0$ in (1.2) and found that the solution's behavior of this case was subtle and more interesting than standard models. In real world, as the authors [3,8,26,27] pointed out, for consideration of the behavior that the interaction between predator and prey affects mainly prey individuals occupying the outermost positions in the herd, since the prey individuals staying on the boundary of the herd are in number proportional to the length of the perimeter of the ground area of the herd, while the perimeter is proportional to the square root of the area, it is reasonable to describe the model with herd behavior by square root term. We know that the impulsive and stage-structure play an important role in making a realistic model, and there is no impulsive