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Internal energy transfer in dynamical behaviour of Timoshenko microarches

Mergen H. Ghayesh [a,*], Hamed Farokhi [b]

[a] School of Mechanical, Materials and Mechatronic Engineering, University of Wollongong, NSW 2522, Australia
[b] Department of Mechanical Engineering, McGill University, Montreal, Quebec, Canada H3A 0C3

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Abstract

The internal energy transfer and modal interactions in the motion characteristics of Timoshenko microarches are investigated numerically. The length-scale parameter is introduced to the strain energy of the system and the equations of motion are obtained via Hamilton's principle based on the modified couple stress theory; these equations are discretized into a set of nonlinear ordinary differential equations through use of the Galerkin scheme. The possibility of the occurrence of modal interactions and internal energy transfers is verified by obtaining the ratios of the linear natural frequencies of the system. The nonlinear response of the system is obtained for the cases with the modal interactions and internal energy transfer.

Keywords: Microbeam; Timoshenko theory; Internal energy transfer; Nonlinear size-dependent behaviour; Modified couple stress theory

1. Introduction

Microbeams [4,9,16], microarches [10], and microplates [3,8,15,18] are present in microswitches, vibration shock sensors, resonators [17], biosensors, microvalves, and electrical microactuators. The experimental results on the motion characteristics of these mechanical elements [11,22,24] showed that they display a size-dependent deformation behaviour which cannot be verified theoretically via classical theories. As a result, new continuum mechanics theories, such as the strain gradient and modified couple stress theories were developed by introducing one or multiple length-scale parameters to the strain energy of the system [20,29,36].

The size-dependent behaviour of *straight* microbeams has been studied by various authors for many years [6,21]. The linear aspects of the problem were analysed, for instance, by Ma et al. [23], who examined the free oscillations of a Timoshenko microbeam based on the modified couple stress theory; Şimşek [33], who investigated the oscillations of an embedded microbeam under the action of a moving microparticle via both analytical and numerical models;

* Corresponding author.
E-mail addresses: mergen@uow.edu.au (M.H. Ghayesh), hamed.farokhi@mail.mcgill.ca (H. Farokhi).

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Nateghi et al. [27], who analysed the size-dependent buckling behaviour of functionally graded microbeams with different boundary conditions; Akgöz and Civalek [1], who employed the modified couple stress theory and the modified strain gradient theory to examine the bending behaviour of an Euler–Bernoulli microbeam; Salamat-talab et al. [32], who investigated the static and dynamic behaviours of a shear deformable functionally graded microbeam. These studies were pursued and extended to nonlinear models. For instance, Moeenfard et al. [25], employed He's homotopy perturbation technique to examine the nonlinear free oscillations of a Timoshenko microbeam. Ramezani [31] contributed to the field by examining the nonlinear free oscillations of a Timoshenko microbeam by means of the method of multiple timescales. Rajabi and Ramezani [30] developed a nonlinear Euler–Bernoulli microbeam model taking into account a surface energy. Asghari et al. [5] analysed the nonlinear free oscillations of a Timoshenko microbeam on the basis of a strain gradient elasticity theory, through use of the method of multiple timescales.

All of the aforementioned precious studies were concerned with *perfectly straight* microbeams. However, realistically, due to imperfections in manufacturing, it is very likely to manufacture an *initially curved* microbeam (i.e., a *microarch*) even if it is intended not to do so. This small imperfection in the microbeam renders the system as a *microarch* with its own unique characteristics and behaviours. Furthermore, in some applications such as in microshutters, microswitches, and bandpass filters, the microbeam is manufactured in a *curved shape intentionally*, forming a microarch. In MEMS applications, microbeams are usually made of polysilicons, which is the case in the present paper.

It is well-known how early investigations of the forced Duffing oscillator led to the discovery of deterministic chaos [35]. The same phenomenon has been observed in a significant number of simple mechanical systems [19,34] including various beam structures under external forcing [26,28]. However, phenomena of this type are not expected to occur in the present study because the ranges of parameters are chosen such that the system operates away from chaos; moreover, the numerical method used in this study is designed for examining the steady-state response of the system and hence is not suitable for determining the presence of any chaotic attractor.

An important issue which is usually ignored in the study of motion characteristics of microscale continuous elements is the energy transfer; the present paper investigates, for the first time, the modal interactions and internal energy transfer in the motion characteristics of a Timoshenko microarch, possessing internal resonances. The equations of motion for the transverse and rotational motions are derived using Hamilton's principle. A high-dimensional discretization is applied on the equations of motion leading into a set of second-order nonlinear ordinary differential equations. A linear analysis is conducted to verify the possibility of internal energy transfer and modal interactions. As we shall see, interesting nonlinear dynamical behaviour is displayed by the system as a result of large displacements in the transverse direction. Moreover, multiple solution branches are generated due to presence of an internal energy transfer mechanism.

2. Equations of motion and Galerkin's scheme

Fig. 1 shows the system under consideration, which is a Timoshenko microarch of length L, cross-sectional area A, area moment of inertia I, Young's modulus E, and shear modulus μ. The hinged–hinged system is subject to a distributed harmonic excitation load per unit length, $F(x)\cos(\omega t)$, in the z direction. x and z represent the axial and transverse directions, respectively; $w(x,t)$ and $\phi(x,t)$ denote the transverse displacement and the rotation of the transverse normal, respectively.

In what follows, the equations of motion are derived based on the following physical and geometrical assumptions: (1) the Timoshenko beam theory is considered, taking into account the shear deformation and rotary inertia; (2) the initial curvature in the transverse direction is represented by $w_0(x)$; (3) a uniform cross-sectional area is assumed along the entire length of the microarch, even after deflection; (4) the longitudinal displacement is neglected [12,13,20,21]; (5) the source of nonlinearity is geometric due to the mid-plane stretching — mid-plane stretching is caused as a result of large displacements compared to the thickness (In particular, when a system undergoes large displacements due to external transverse excitation, the centreline of the microarch tends to stretch, causing a tensile axial stress, and hence changing the stiffness of the system nonlinear wise).

In what follows, the modified couple stress theory is employed to obtain the equations of motion of the microarch. The modified couple stress theory, developed by Yang et al. [36], facilitates the application of the classical couple stress theory by neglecting the stretch and dilation gradients, and considering the curvature tensor as deformation measures in addition to the classical strain measures; hence, it consists of only one material length-scale parameter as well as two classical material constants.