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A new Lorenz-type hyperchaotic system with a curve of equilibria☆

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Highlights

- Find a new 4D hyperchaotic system with a curve of equilibria.
- Exhibit complex dynamics such as hyperchaotic, chaotic and quasi-periodic.
- Find lots of singular degenerate heteroclinic cycles in the new hyperchaotic system.
- Find four different kinds of coexisting attractors, which are reported rarely.
- Our paper may give a contribution in better understanding 4D Lorenz-type system.

Abstract

Little seems to be known about hyperchaotic systems with a curve of equilibria. Based on the classical Lorenz system, this paper proposes a new four-dimensional Lorenz-type hyperchaotic system which has a curve of equilibria. This new system can generate not only hyperchaotic attractors but also chaotic, quasi-periodic and periodic attractors, as well as singular degenerate heteroclinic cycles. Of particular interest is the observation that there are four types of coexisting attractors of this new hyperchaotic system: (i) chaotic attractor and quasi-periodic attractor, (ii) chaotic attractor and singular degenerate heteroclinic cycle, (iii) periodic attractor and singular degenerate heteroclinic cycle, and (iv) different periodic attractors. Furthermore, many singular degenerate heteroclinic cycles are found, which may lead to complex dynamics of hyperchaotic system with a curve of equilibria.

Keywords: Lorenz system; Hyperchaos; Curve of equilibria; Coexisting attractor; Singular degenerate heteroclinic cycles

1. Introduction

Since the first formulation of a model of a chaotic oscillator, i.e. the Lorenz model [11], chaos theory has established itself as an important part of contemporary science, and chaotic phenomena have been demonstrated to arise in a broad variety of fields from mathematics and physics through many areas of engineering to biology and the social sciences [2–4,14,19,18,13,5]. A number of important chaotic systems, such as the Chen [1], the Lü [12] and the Yang systems [18] are known to display behaviors related to those of the Lorenz system. As a kind of behavior that is more complex than chaos, hyperchaos is characterized by at least two positive Lyapunov exponents. Originally suggested by Rossler in 1979 [15], this type of dynamics continues to attract considerable interest in many fields of science and technology.

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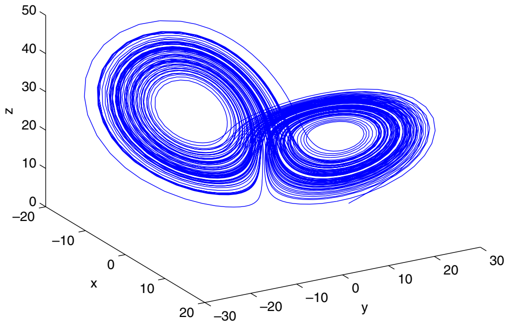

Fig. 1. Chaotic attractor of Lorenz system (1): $a = 10$, $b = 8/3$, $c = 28$.

Recently, lots of researches are involved in categorizing periodic and chaotic attractors as either self-excited or hidden [8,10,9,17]. Most known chaotic and hyperchaotic systems have one to three equilibria, such as the above mentioned systems [11,18,1,12,15], their attractors are self-excited attractor which has a basin of attraction that is associated with an unstable equilibrium. In contrast, the hidden attractor, its rigorous mathematical definition is suggested first by Leonov and Kuznetsov [9], has a basin of attraction that does not intersect with small neighborhoods of any equilibria. Obviously, any dissipative chaotic system with only stable equilibria or even with no equilibrium must have a hidden attractor [9,17,16,6].

In chaos theory, the equilibrium of a dynamical system is significant for understanding its complex dynamics. Up to now, most chaotic and hyperchaotic systems are reported to have just a limited or countable number of isolated equilibria. Recently, Jafari and Sprott introduced a new category of chaotic systems with hidden attractors: three-dimensional systems with a line equilibrium [7]. For four-dimensional autonomous system, to the best of our knowledge, little seems to be known about the hyperchaotic system with a curve of equilibria [20], and only a few dynamical behaviors are observed for such systems. It is therefore interesting to ask whether four-dimensional autonomous systems with a curve of equilibria can display complex dynamics such as hyperchaotic, chaotic, quasi-periodic and periodic attractors, singular degenerate heteroclinic cycles, and coexisting of different attractors. This question will be addressed in the present paper.

With the help of feedback control techniques and based on the classical three-dimensional Lorenz system, this paper proposes and analyzes a new four-dimensional Lorenz-type hyperchaotic system with a curve of equilibria. This new hyperchaotic system, which can display hyperchaotic, chaotic, periodic and quasi-periodic dynamics, is investigated through mathematical analysis and numerical simulation including Lyapunov exponents, Poincaré image and bifurcation diagram, etc. Meanwhile, many singular degenerate heteroclinic cycles are found, which may lead to complex dynamics of hyperchaotic systems with a curve of equilibria. Especially, there are four types of different coexisting attractors of this new hyperchaotic system: (i) chaotic attractor and quasi-periodic attractor, (ii) chaotic attractor and singular degenerate heteroclinic cycle, (iii) periodic attractor and singular degenerate heteroclinic cycle and (iv) different periodic attractors.

2. New hyperchaotic system with a curve of equilibria

In 1963, E.N. Lorenz proposed the first chaotic system, i.e. the famous Lorenz system [11]

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - y - xz, \\ \dot{z} = -bz + xy, \end{cases} \tag{1}$$

where a, b and c are real parameters and dot denotes derivative with respect to time t. When the parameters $a = 10$, $b = 8/3$ and $c = 28$, which are known as the classical parameters of Lorenz system, system (1) has a chaotic attractor that coexists with a saddle and a pair of symmetrical saddle–focus equilibria. The phase portrait of Lorenz attractor is shown in Fig. 1.