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A simple method for constructing integro spline quasi-interpolants☆

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Abstract

In this paper, we study a new method for function approximation from the given integral values over successive subintervals by using cubic B-splines. The method does not need any additional end conditions and it is easy to be implemented without solving any system of linear equations. The method is able to approximate the original function and its first and second-order derivatives over the global interval successfully. The approximation errors are well studied. Numerical results illustrate that our method is very effective.

Keywords: B-spline; Quasi-interpolant; Integral value; End conditions

1. Introduction

Let f be an unknown univariate real-valued function over $\boldsymbol{I} = [a, b]$, and that

$$\Delta := \{a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b\}$$

be a uniform partition of $\boldsymbol{I}$ into n subintervals. In traditional interpolation/approximation problems by spline functions, we assume that the function values at knots are given, and we use them to construct the spline function. In this paper, these function values are not given, and we assume that the integral values over subintervals $[x_j, x_{j+1}]$, $j = 0, 1, \ldots, n-1$ are given. We denote by $\mathcal{I}_j$ these integrals, i.e.

$$\mathcal{I}_j = \int_{x_j}^{x_{j+1}} f(x)dx, \quad j = 0, 1, \ldots, n-1. \tag{1}$$

The problem is to approximate f and its derivatives by using the given data in (1). This kind of problem arises frequently in mechanics, mathematical statistics, numerical analysis, electricity, environmental science, climatology, oceanography and so on, see [1,2,8,11–13,20]. Moreover, spline functions used for the reproduction of integral values are often referred to as histosplines and has been studied in [3,5,9,16,19] and references therein.

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Recently, various methods have been developed in the literature for constructing f. A regularization method was proposed in [12]. This method obtain an approximate function for f by solving a minimization problem, which was defined by a cost functional with an additional regularization parameter that must be chosen properly. The approximate function was proved to be the unique quadratic spline satisfying some special conditions over $[a, b]$ with partition Δ. Their order of convergence is only 2 and dependent not only on f but also on a regularization parameter. Some other authors applied spline technique directly to study the problem. A quartic integro spline interpolation method was used in [8]. It needed to solve a linear system with a full $(2n + 2) \times (2n + 2)$ coefficient matrix. Moreover, it also needed another four additional boundary conditions ($f(x_0)$, $f'(x_0)$, $f(x_n)$ and $f'(x_n)$ or other sets of boundary conditions) besides the data in (1). In fact, the method was not effective. In [1], Behforooz first introduced the motivation of integro cubic splines to approximate the function and analyzed the approximation problem with various end conditions. He showed that when the first derivative representation and first derivative end conditions are used to construct the splines, it is necessary to solve a tridiagonal system of linear equations. Besides two end conditions, also one additional/or third end condition was required. He pointed out that to construct the integro cubic splines in terms of the second derivative with any end conditions one has to solve a system of linear equations with a full matrix of higher order. Later, a quintic integro spline interpolation method was given in [2] based on the "quintic Hermite–Berkhoff interpolation polynomial". This method required to solve three linear systems by using seven additional boundary conditions ($f(x_0)$, $f'(x_0)$, $f'(x_1)$, $f'(x_{n-1})$, $f'(x_n)$, $f'''(x_0)$ and $f'''(x_n)$). The method is very complicated. In general, these methods have two main drawbacks. On the one hand, they need some additional end conditions besides the given integral values in (1), and they have a higher computational cost. On the other hand, these methods have not studied the approximations of the derivatives of f. The features limit the applications of these methods. Recently, a local integro cubic spline method was given in [20]. It was able to reconstruct $f^{(k)}$, $(k = 0, 1, 2)$ with approximation orders on $\mathcal{O}(h^{4-k})$ respectively. The method did not need any additional data and has lower computational complexity. Hence, it has more wide applications.

In this paper, we propose to use a cubic spline quasi-interpolant for the reconstruction of a function f by using the given data in (1). The main advantage of this approach is that it does not need any additional data. Moreover, the spline quasi-interpolant has a direct construction without solving any system of linear equations, unlike what happens with interpolants. Furthermore, as illustrated in Section 5, an optimal approximation order is obtained with a smaller infinity norm than the one given in [20].

The paper is organized as follows. In Section 2, we give some preliminary results on spline quasi-interpolants and we construct a quasi-interpolant based on cubic polynomial B-splines. In Section 3, we approach the function values with a linear combination of the given integro values in (1) and we construct our approximation operator. Section 4 is devoted to study the approximation errors of this local operator and its derivatives. In Section 5, we illustrate the performance of the method with some numerical tests. Finally, in Section 6, we give a conclusion.

2. Quasi-interpolant based on cubic B-splines

In this section we recall some types of spline quasi-interpolants and we construct a quasi-interpolant based on cubic polynomial B-splines which will be used in Section 4.

We denote by $\mathcal{S}_d(\boldsymbol{I}, \mathcal{X}_n)$ the space of splines of degree d and of class $\mathcal{C}^{d-1}$ on the uniform partition $\mathcal{X}_n = \{x_i = a + ih,\ i = 0, 1, \ldots, n\}$ with mesh length $h = \frac{b-a}{n}$. A basis of this space is $\{B_j^d,\ j \in \mathcal{J}\}$ with $\mathcal{J} = \{1, 2, \ldots, n+d\}$. With these notation, $\operatorname{supp}(B_j^d) = [x_{j-d-1}, x_j]$ and $\mathcal{N}_j = \{x_{j-d}, x_{j-d+1}, \ldots, x_{j-1}\}$ is the set of the interior knots in the support of B_j^d. As usual, we add multiple knots at the end points $x_{-d} = \cdots = x_{-1} = x_0 = a$ and $b = x_n = x_{n+1} = \cdots = x_{n+d}$.

Univariate spline quasi-interpolants can be defined as operators of the form

$$\mathcal{Q}_d f := \sum_{j=1}^{n+d} \mu_j(f) B_j^d,$$

where μ_j are local linear functionals which are in general of one of the following types:

(i) Differential type: $\mu_j(f)$ is a linear combination of values of derivatives of f, of order at most d, at some points in the neighborhood of $\operatorname{supp}(B_j^d)$ (see e.g. [6,7]). The associated quasi-interpolant is called a differential quasi-interpolant (abbr. DQI).