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Original article

Dynamics in piecewise linear and continuous models of complex switching networks

Pabel Shahrear [a,*], Leon Glass [a], Roy Wilds [b], Rod Edwards [c]

[a] *Department of Physiology, 3655 Promenade Sir William Osler, McGill University, Montreal, Quebec, Canada H3G 1Y6*
[b] *Rugged Data, Ottawa, Canada*
[c] *Department of Mathematics and Statistics, University of Victoria, P.O. Box 3060, STN CSC, Victoria, BC, Canada V8W 3R4*

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Abstract

Activities of genes are controlled in a combinatorial fashion by the concentrations of chemical called transcription factors. We model this type of network by piecewise linear differential equations formed by embedding a logical switching network in a differential equation. We generate continuous nonlinear equations by replacing the step function discontinuities in the piecewise linear equations by sigmoidal control functions. As the sigmoidal functions become steep, the continuous equations approach piecewise linear differential equations. We carry out numerical studies of the continuous and piecewise linear equations for a 4-dimensional example with particularly interesting and complex behavior, showing that the dynamics in the continuous equation approaches those in the piecewise linear equation as the sigmoids become steep.

Keywords: Piecewise linear equations; Chaotic dynamics; Genetic networks

1. Introduction

Complex networks are ubiquitous in biological systems. The development of methods that can be used to study the properties of complex networks constitutes an important direction for research. One approach is to develop logical models that capture key features of the structure of biological networks [21,14]. These logical models then provide a basis for more realistic classes of models that can be obtained by embedding the logical structure in differential equations [11] or by assuming stochastic updating [3]. There is now a large body of work that demonstrates the utility of studying the logical structure underlying biological networks. Several recent papers can be consulted for specific examples and further references to earlier work [1,2].

The current paper deals with dynamics in differential equations with an embedded logical structure. Given a logical structure represented by a truth table, for example see Table 1, it is possible to construct a state transition graph, for example see Fig. 1. The nodes in the state transition graph correspond to volumes of state space and the directed edges between nodes show allowable transitions [11]. The state transition graph can be useful in predicting dynamics based solely on the logical structure. For example, nodes with an out-degree equal to zero will lead to stable steady states in

* Corresponding author. Tel.: +1 5143988224.
E-mail address: pabelshahrear@yahoo.com (P. Shahrear).

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Table 1
Truth table for a 4-dimensional Boolean network.

$(X_1\, X_2\, X_3\, X_4)$	$(b_1\, b_2\, b_3\, b_4)$
(0 0 0 0)	(1 1 1 0)
(0 0 0 1)	(0 0 0 0)
(0 0 1 0)	(0 1 1 0)
(0 0 1 1)	(1 0 0 1)
(0 1 0 0)	(0 1 1 0)
(0 1 0 1)	(0 1 1 1)
(0 1 1 0)	(1 1 1 0)
(0 1 1 1)	(1 0 1 0)
(1 0 0 0)	(1 0 1 1)
(1 0 0 1)	(0 1 0 1)
(1 0 1 0)	(1 1 1 1)
(1 0 1 1)	(1 0 0 1)
(1 1 0 0)	(0 0 0 1)
(1 1 0 1)	(0 1 0 1)
(1 1 1 0)	(1 1 0 1)
(1 1 1 1)	(1 0 0 1)

the associated differential equations and attracting cycles in a state transition diagram imply stable cycles in a class of piecewise linear homologous ordinary differential equations [13,5]. Lu and Edwards extended this work by describing new structural conditions for the state transition graph that will lead to a broader class of stable cycles [17]. Chaotic dynamics in particular 4-dimensional piecewise linear equations have been analyzed [19,6]. Although progress has made on understanding design principles that lead to complex dynamics for networks in which one node in the state transition graph lies on two different cycles [18,7], we do not yet have strong conditions for chaotic dynamics based on the logical structure.

We have been interested in the connection between dynamics in piecewise linear equations and their continuous homologues. An early paper demonstrated methods to generate continuous differential equations modeling genetic

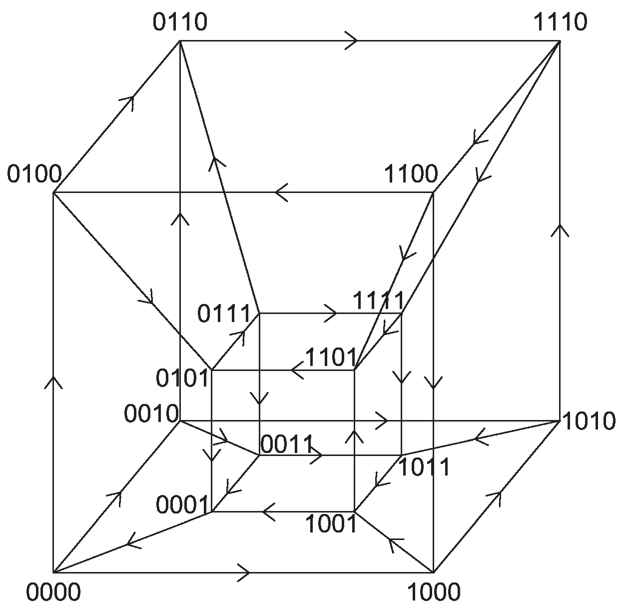

Fig. 1. A hypercube representation of the network in Table 1. Each vertex represents an orthant of state space, and the edges between the vertices represent the allowed transitions between orthants in the associated differential equation.