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Numerical simulation of the modified regularized long wave equation by split least-squares mixed finite element method

Fuzheng Gao*, Feng Qiao, Hongxing Rui

School of Mathematics, Shandong University, Jinan, Shandong 250100, China

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Abstract

A kind of modified regularized long wave (MRLW) equation, with some initial conditions, is solved numerically by a split least-squares mixed element method (SLSMEM), which can be split into two independent symmetric positive definite sub-schemes and solved separately. This method is useful for obtaining numerical solutions with high degree of accuracy. Numerical examples show that the SLSMEM satisfies conservation laws.

Keywords: MRLW equation; Least-squares; Mixed element; Conservation laws; Solitary waves

1. Introduction

Sobolev equation is a classical nonlinear partial differential equation, which includes a third order mixed derivative with respect to time and space, whose model equation is

$$u_t + f(u)_x - \gamma u_{xx} - \mu u_{xxt} = 0. \tag{1}$$

The problem (1) arises from flow of fluids through fissured rock [8], migration of moisture in soil [33], thermodynamics [35] and other applications. It is used to describe wave motion in media with nonlinear wave steepening and balancing with dispersion and diffusion, which are important not only in hydrodynamics but also in many other disciplines of engineering and science.

The best example is regularized long wave (RLW) or Benjamin–Bona–Mahony (BBM) equation, MRLW equation, and generalized regularized long wave (GRLW) equation. In fact, when $f(u) = u + \frac{\delta}{2}u^2$ and $\gamma = 0$, (1) can be converted into the following RLW equation

$$u_t + u_x + \delta u u_x - \mu u_{xxt} = 0. \tag{2}$$

When $f(u) = u + 2u^3$ and $\gamma = 0$, (1) can be rewritten as

$$u_t + u_x + 6u^2 u_x - \mu u_{xxt} = 0, \tag{3}$$

which is called modified regularized long wave (MRLW) equation.

* Corresponding author.
E-mail address: fzgao@sdu.edu.cn (F.Z. Gao).

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RLW equation (2) was originally introduced to describe the behavior of undular bore by Peregrine [29]. δ and μ are positive constants and $u(x,t)$ is amplitude of long wave at position x and time t. Indeed, RLW equation (2) and MRLW equation (3) are all special cases of generalized long wave (GRLW) equation, which has form

$$u_t + u_x + \delta u^r u_x - \mu u_{xxt} = 0. \tag{4}$$

A large number of numerical methods have been presented for solving this kind of equations from (1) to (4), e.g., standard FEM [6,36,12,13,26,1,21,17,22], Petrov–Galerkin method [11,7], mixed FEM [19,28,20] and local discontinuous Galerkin (LDG) FEM [14,37]; finite difference methods (FDM) [2,27,23]; Adomian decomposition methods [3,24]; homotopy perturbation method [4]; He's variation iteration method [25,34]; or generalized difference method (GDM) [9].

Recently, we have proposed split (or split characteristic) least-squares mixed FEMs in [15,16] for the Sobolev equations and derived optimal error accuracy results. In this paper, we focus on studying the conservation properties of the proposed scheme numerically.

The main work in this paper is twofold. We first decompose the original problem (4) into two kinds of ordinary differential equations, one includes time derivative and the other includes space derivative. The other highlight is the derived scheme can be split into two independent symmetric positive definite sub-schemes and solved separately based on the idea in [15,16,31,32,18]. Numerical examples show that the proposed scheme is conservative.

The outline of this article is as follows. Section 1 is introduction. In Section 2, analytical solution and invariants of MRLW equation are presented. In Section 3 some preparatory works are considered, and SLSMEM for problem (3) is formulated. In Section 4, we give three numerical examples to verify the feasibility of the proposed scheme. Finally, some remarks are given in Section 5.

Throughout this paper, the notations of standard Sobolev spaces $L^2(\Omega)$, $H^k(\Omega)$ and associated norms $\|\cdot\| = \|\cdot\|_{L^2(\Omega)}$, $\|\cdot\|_k = \|\cdot\|_{H^k(\Omega)}$ are adopted as those in [10,5].

2. Analytical solution and invariants of the MRLW equation

As similar to [25], we consider MRLW equation (3) with boundary conditions $u \to 0$ as $x \to \pm\infty$. Given computational region $\Omega = [a,b]$, The exact solution of (3) is same to [25] as follows

$$u(x,t) = \sqrt{c}\,\mathrm{sech}[K(x-(c+1)t-x_0)], \tag{5}$$

where c and x_0 are arbitrary constants and $K = \sqrt{\frac{c}{\mu(c+1)}}$. Initial value $u(x,0) = u_0(x)$ and boundary value can be derived by analytic solution (5).

The MRLW equation with periodic boundary conditions or compact support has invariants of the form [4,17,22,24] and [25].

$$\begin{cases} I_1 = \int_a^b u\,\mathrm{d}x, \\ I_2 = \int_a^b (u^2 + \mu u_x^2)\,\mathrm{d}x, \\ I_3 = \int_a^b (u^4 - \mu u_x^2)\,\mathrm{d}x, \end{cases} \tag{6}$$

which are used to measure whether (or how) the numerical scheme is useful, especially for problems with unavailable analytic solution and during the interaction of solitons.

3. The split least-squares mixed-element method

Based on the ideas of [15,16], we present fully-discrete scheme, in which time level is updated by backward Euler time-marching, for problem (4).