Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 109 (2015) 174–185
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

Spatial patterns through Turing instability in a reaction–diffusion predator–prey model

Lakshmi Narayan Guin*

Department of Mathematics, Visva-Bharati, Santiniketan-731 235, West Bengal, India

Received 22 November 2013; received in revised form 28 September 2014; accepted 6 October 2014
Available online 18 October 2014

Abstract

Pattern formation in nonlinear complex systems is one of the central problems of the natural, social and technological sciences. In this paper, we consider a mathematical model of predator–prey interaction subject to self as well as cross-diffusion, arising in processes described by a system of reaction–diffusion equations (coupled to a system of ordinary differential equations) exhibiting diffusion-driven instability. Spatial patterns through Turing instability in a reaction–diffusion predator–prey model around the unique positive interior equilibrium of the model are discussed. Furthermore, we present numerical simulations of time evolution of patterns subject to self as well as cross-diffusion in the proposed spatial model and find that the model dynamics exhibits complex pattern replication in the two-dimensional space. The obtained results unveil that the effect of self as well as cross-diffusion plays an important role on the stationary pattern formation of the predator–prey model which concerns the influence of intra-species competition among predators.

Keywords: Reaction–diffusion systems; Self and cross-diffusion; Instability; Pattern formation

1. Introduction

Ecological models have been in the focus of ecological science as predation of interacting species affects population dynamics significantly. Studies on stability mechanism and the theory of spatial pattern formation through diffusion-driven instability [39] of a system of interacting populations in which a nonlinear system is asymptotically stable in the absence of diffusion but unstable in the presence of diffusion play significant role in mathematical ecology, embryology and other branches of science [3–5,18,21,24,25,28]. Spatial patterns modify the temporal dynamics and stability properties of population densities at a range of spatial scales, their effects must be incorporated in temporal ecological models that do not represent space explicitly. And the spatial component of ecological interactions has been identified as an important factor in how ecological communities are shaped [7,19,23,26,40]. In the predator–prey system models, the interaction between the predator and the prey is the reaction item and the diffusion item comes by reason of pursuit-evasion phenomenon—predators pursuing prey and prey escaping predators [1,37,38]. In such

* Tel.: +91 3463261029; fax: +91 3463261029.
E-mail address: guin_ln@yahoo.com.

http://dx.doi.org/10.1016/j.matcom.2014.10.002

a system, there is a tendency that the prey stays away from the predators and the escape velocity of the preys may be taken to be proportional to the dispersive velocity of the predators. In the same manner, there is a tendency that the predators will get closer to the preys and the chase velocity of predators may be considered to be proportional to the dispersive velocity of the preys [32]. Therefore the problem of cross-diffusion arises, which was proposed first by Kerner [14] and first applied to competitive population systems by Shigesada et al. [30].

The effect of diffusion on the spatiotemporal predator–prey model has been investigated by many scientists to their insightful work [16,20,29,41]. Recently, the effect of self as well as cross-diffusion in reaction–diffusion systems has received much attention by both ecologists and mathematicians [6,9,15,31,33]. The term self-diffusion which implies the per capita diffusion rate of each species is influenced only by its own density, i.e. there is no response to the density of the other one. On the other hand, cross-diffusion implies the per capita diffusion rate of each species which is influenced not only by its own but also by the other ones density. The value of the cross-diffusion coefficient may be positive, negative or zero. The positive cross-diffusion coefficient denotes the movement of the species in the direction of lower concentration of another species while the negative cross-diffusion coefficient for one species tends to diffuse in the direction of higher concentration of another species.

In the studies of pattern formation on spatiotemporal predator–prey model with prey-dependent Holling type II functional response, little attention has been paid here to the effect of self as well as cross-diffusion, which is an extension work of the following Bazykin's [2] model:

$$\frac{du}{dt} = ru\left(1 - \frac{u}{k}\right) - \frac{auv}{u+c} = f_1(u, v), \tag{1.1a}$$

$$\frac{dv}{dt} = -dv + \frac{buv}{u+c} - hv^2 = f_2(u, v), \tag{1.1b}$$

$$u(0) > 0, \qquad v(0) > 0, \tag{1.1c}$$

where u, v denote prey and predator population size respectively at any instant of time t, and all the parameters in uniform environment viz. r, k, a, b, c, d, h are positive. The parameter r designates the intrinsic growth rate of the prey species. Similarly k denotes the carrying capacity of the prey species, a, the predation rate or capturing rate of prey by predator, b, the maximal predator growth rate, c, the interference coefficient of the predator, d, the predator natural mortality rate and h represent the predator intra-species competition.

The diffusion effect affecting the reaction–diffusion predator–prey system is one of these influent elements, modifying qualitative stability and quantitative aspects of the spatiotemporal dynamics in diffusive models. As predator–prey interactions are inherently prone to oscillations, it is therefore obvious to investigate this phenomenon as a potential mechanism for the creation of spatial patterns through Turing instability induced by self as well as cross-diffusion. However, only a few earlier works have been devoted to this issue [6,8,9,33], which is an important objective in the present work. Our present model for spatially extend systems is devoted to explain spatiotemporal behaviour of interacting populations through complex pattern replication (viz. stripe, spotted, labyrinthine or spot–stripe mixtures patterns), which concerns the influence of intra-specific competition among predators.

Intra-specific competition is a particular form of competition in which members of the same species compete for limited resources in an ecosystem (e.g. food, water, space, light, nutrients, mates or any other resource which is required for survival). This can be contrasted with inter-specific competition, in which different species compete for a shared resource. Members of the same species have very similar resources requirements whereas different species have a smaller contested resource overlap, resulting in intra-specific competition generally being a stronger force than inter-specific competition. Whenever populations of a species are crowded, intra-specific competition is intense. Intra-specific competition is a major factor affecting the carrying capacity of a population. In brief, intra-specific competition refers to a decrease in reproduction or an increase in death rate with an increase in species density [2,10–13]. In the present paper, we focus mainly on the dynamics of pattern formation in the predator–prey model with self as well as cross-diffusion effect. One of the objectives of this study is to explore the effect of diffusion and cross-diffusion coefficients on spatio-temporal dynamics in a prey-dependent predator–prey model and the novelty of this study lies with the inclusion of intra-specific competition among the predators. The rest of this paper is organized as follows. In Section 2, we introduce the model with diffusion and give the existence and feasibility criteria of the unique interior equilibrium point of (1.1). In Section 3, we discuss the results of Turing pattern formation via numerical simulations. Finally, conclusions and remarks are presented in Section 4.