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Tuning the average path length of complex networks and its influence to the emergent dynamics of the majority-rule model

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Abstract

We show how appropriate rewiring with the aid of Metropolis Monte Carlo computational experiments can be exploited to create network topologies possessing prescribed values of the average path length (APL) while keeping the same connectivity degree and clustering coefficient distributions. Using the proposed rewiring rules, we illustrate how the emergent dynamics of the celebrated majority-rule model are shaped by the distinct impact of the APL attesting the need for developing efficient algorithms for tuning such network characteristics.

Keywords: Complex networks; Average path length; Prescribed network characteristics; Monte Carlo simulations; Majority-rule model

1. Introduction

The strong interplay between the emergent complex dynamics of many real-world systems and the underlying topology of the networks that pertain to their structure has been illustrated by many studies: the dynamics of electrical power transmission systems including cascade failures leading to blackouts [54], the evolution of the world wide web, various social phenomena such as mimesis and herding [55], brain cognitive, neurological disorders and motor functions [8,11,54] are typical paradigms of such cases.

Thus, the modeling and the systematic investigation of the topological properties of complex networks are of great importance. Towards this aim, various algorithms for generating networks aspiring to approximate the actual ones have been proposed [2,46,57].

In order to approximate such real-world network structures, Watts and Strogatz [57] (WS) constructed a network model with a variable connectivity degree possessing "small-world" properties interpolating between regular ring lattices and random regular networks (RRN). Small-world networks are highly clustered with small path lengths. The most famous experiment which describes the concept of "small-worldness" was conducted by the physiologist Milgram [44]. He sent 160 letters to residents of the city Omaha in Nebraska asking them to post a letter to a friend.

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The shipment had only one term: "do not try to send the mail directly if you don't know the recipient; instead, post the mail to one friend of yours who you believe knows the recipient". The letters, which were finally received, had been posted about six times on average. This phenomenon is also known as six degrees of separation (also known as "six degrees of Kevin Bacon" [57]). Other networks, such as the Internet, include nodes with overwhelmingly more connections compared to other nodes (for instance yahoo, google and amazon are nodes with a huge number of connections). This type of networks is usually characterized by a power-law distribution of degrees i.e. $P(k) = ck^{-\gamma}$ and are called scale-free. Barabasi and Albert [6] in their seminal paper proposed an algorithm for generating networks with power-law degree distributions. These networks can capture the power-law characteristic which pertains to the structure of many real-world networks, yet the clustering coefficient decays very fast with network size; therefore, failing to approximate larger clusters as these are observed in many real life structures [35]. Furthermore, there are many social networks whose structure deviates from the power-law degree distribution or small-worldness (most often exhibiting skewed degree distributions [12]). In addition, the topological characteristics of many networks may change over time. For example in [49], it is shown that in the contact social network of a disease transmission at an American high school the clustering coefficient remained almost constant over a wide range of contact durations while the average path length doubles.

In order to generate networks structures that can capture the empirically observed ones, research efforts have been focused on developing algorithms for generating network topologies with prescribed characteristics. In [28], the authors propose a network-growing algorithm combining the properties of both scale-free networks and small-world networks. The value of the clustering coefficient is driven by manipulating the formation of triades. Serrano and Boguna [50] present a network-growing algorithm for controlling both the degree distribution and the clustering coefficient. Their algorithm is based on the so-called configuration model which is used to generate pre-assigned degree distributions. The most typical representative of this category is the celebrated Erdòs–Rényi algorithm [20]. Volz [56] uses a Markov chain Monte Carlo technique to generate both a given degree distribution and a clustering coefficient. Maslov and Sneppen [41] use a rewiring technique to produce random networks, with a given connectivity degree, in the case of the interactions of nuclear proteins. Kim [33] introduces an algorithm based on a Monte Carlo simulation at both zero and finite temperatures to control the clustering coefficient of a given network. In [24], the authors propose an algorithm for generating small-world networks with tunable assortative coefficient. Leary et al. [38] present an algorithm for controlling the degree distribution by altering the preferential attachment step in the Barabàsi and Albert algorithm. Exploiting the algorithm of Holme and Kim, they were able to produce different degree distributions with different clustering coefficients. Badham and Stocker [5] propose an algorithm for adjusting three properties of networks, namely the degree distribution, the clustering coefficient and the assortativity. In [21], the authors propose an optimization method for constructing a network with prescribed degree-dependent clustering.

In this work, we propose appropriately chosen rewiring rules that can be used to systematically construct consistent to APL network structures at will, yet maintaining the degree and clustering distributions untouched. To demonstrate the approach, we constructed networks (starting from small-world networks which served as our initial configurations derived using the WS algorithm [57]), with prescribed values of APLs. At this point we should note that upon the construction of a small-world or a scale-free network using an algorithm such as the one proposed in [57] or in [2], the APL cannot be in principle prescribed.

Furthermore, we show how different topologies as these are obtained by adjusting the value of the APL can dramatically shape the emergent dynamics of network-based models. For our illustrations, we chose a significant representative of such cases: the majority-rule model. Majority-rule models have been extensively used to simulate and gain a better understanding of the behavior of many complex systems ranging from epidemic spread dynamics [9,10,31,42] and opinion formation and voter/election dynamics [14,27,37,53] to culture and language dynamics [4,16,18,19,40], crowd flow design and management [25,26,29,48], diffusion of news and innovations [23,39,58], and ecology and neuroscience [36,51].

2. Tuning the average path length of a complex network

The clustering coefficient and the APL of a network are two attributes that contain significant information concerning its topological structure. The APL, say L, is a global property indicating the average number of steps required to reach any two nodes. It is defined as the mean value of all shortest paths between any two nodes, i.e. $L = \frac{\sum d_{i \leftrightarrow j}}{\frac{N(N-1)}{2}}$,