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Original article

An unbounded stabilization problem of bilinear systems

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Abstract

In this paper, we propose a family of feedback controls that guarantee the strong stabilization of unbounded parabolic bilinear systems, where the operator of control is supposed unbounded in the sense that it is bounded from the state space into some extension. An explicit decay estimate is established. An illustrating example is given.

Keywords: Unbounded bilinear system; Feedback control; Strongstabilization; Decay estimate; Admissibility

1. Introduction

In this paper we consider infinite-dimensional bilinear control systems of the form:

$$\frac{dz(t)}{dt} = Az(t) + v(t)Bz(t), \quad z(0) = z_0 \in H, \tag{1}$$

where the state space H is a real Hilbert with inner product $<, >$ and corresponding norm $\| \ \|$, A generates a strongly continuous semigroup of contractions $(S(t))_{t\geq 0}$ on H and B is an unbounded linear operator from H to a Banach extension X of H with a continuous injection $H \hookrightarrow X$, so that $\|By\|_X \leq M\|y\|$, $\forall y \in H$, for some $M > 0$. This is the case, when for example the control is exercised over the boundary or at a point of the geometrical domain of the system. The scalar function $v(.)$ denotes the control.

We shall suppose that A admits an extension, still denoted by A, which generates a strongly continuous semigroup on X, also denoted by $(S(t))_{t\geq 0}$, so we have

$$\|S(t)By\|_X \leq M \quad e^{\omega t} \quad \|B\|_{L(H,X)} \quad \|y\|, \quad \forall y \in H, \tag{2}$$

for some constants $M > 0$, $\omega \geq 0$.

Bilinear controls are essential in modeling reaction-diffusion convection processes controlled by means of so-called catalysts that can accelerate or decelerate the reaction at hand, which is the case for various chemical or biological chain reactions [22] (see for instance [23,26,30]). In many control problems where (bio)chemical reactions and transport phenomena occur, control actions may give rise to the unboundedness aspect of the operator of control in the obtained bilinear model. This is the case for example if the control take place at the boundaries of the system's evolution

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domain. This may also occur in the case of controls with time delay. The authors in [5,7,9] study the "unboundedness" of the control operators based on the notion of "admissible" operators and appropriate regularity assumptions (see also [4,9,16,19]).

The operator B is said to be (p, q)-admissible for $1<p, q<+\infty$ such that $\frac{1}{p}+\frac{1}{q}\leq 1$, if for any $t>0$, the integral $\int_0^t v(s)S(t-s)By(s)\quad ds$ lies in H and depends continuously on $v\in L^p(0,+\infty;\mathbb{R})$ and $y\in L^q(0,+\infty;H)$. In the other word the following bilinear operator $\Phi_t : L^p(0,+\infty;\mathbb{R})\times L^q(0,+\infty;H)\longrightarrow H$ defined by $\Phi_t(y,v)=\int_0^t v(s)S(t-s)By(s)\quad ds$ is continuous, and let C_t denotes its norm. In this context, if B is (p, q)-admissible then the open loop system (1) admits a unique mild solution $z\in\mathcal{C}([0,+\infty[, H)$ (see [5,19]).

In [7], the author treats the well-posedness and stability of the closed-loop system:

$$z'(t)=Az(t)+f(< Bz(t), z(t) >)Bz(t),\quad z(0)=z_0, \tag{3}$$

under the following assumptions:

(a) $f:\mathbb{R}\longrightarrow\mathbb{R}$ is nonnegative, nondecreasing and continuous,
(b) A generates a strongly continuous semigroup of contractions on H,
(c) $D((I-A)^1/2)=D((I-A^*)^1/2)$ with equivalent norms,
(d) $< (I-A)y, y > \geq C\|(I-A)^1/2y\|^2,\quad \forall y\in\mathcal{D}(A)$ (for some $C>0$),
(e) B is a positive self-adjoint and bounded from the subspace $V=\mathcal{D}((I-A)^1/2)$ of H to its dual space.

In [17], a quadratic control has been proposed to get the estimate $\|z(t)\|=O(\frac{1}{\sqrt{t}})$ which cannot be, in general, improved with quadratic controls.

In this paper, we study the polynomial stabilizability of the system (1) and we give an explicit estimate of the stabilized state.

The plan of the paper is as follows: In the next section, we present an appropriate decomposition of the system (1) via the spectral properties of A, and we apply this approach to study the stabilization problem of (1). In the third section, we give an illustrating example.

2. Stabilization problem

2.1. Decomposition of the system and well-posedness

In what follows, we suppose that A is self-adjoint with compact resolvent, so there are only finitely many eigenvalues $(\lambda_i)_{1\leq i\leq N}$ such that $\lambda_i\geq-\eta$ for some $\eta>0$, each with finite dimensional eigenspace [21,33], and hence the space H can be decomposed according to

$$H=H_u\oplus H_s, \tag{4}$$

where $H_u=Vect(\varphi_i, 1\leq i\leq N)$, $H_s=Vect(\varphi_i, i>N)$ and for all $i\geq 1$, φ_i is an eigenvector associated to the eigenvalue λ_i and we have

$$\|S_s(t)\|\leq M_1\exp(-\eta t),\quad \forall t\geq 0\quad (\text{for } some\ M_1>0), \tag{5}$$

where $S_s(t)$ denotes the restriction of the semigroup $S(t)$ in H_s.

Let us consider the two systems:

$$\frac{dz_u(t)}{dt}=A_uz_u(t)+v(t)B_uz_u(t),\quad z_u(0)=z_{u0}\in H_u, \tag{6}$$

$$\frac{dz_s(t)}{dt}=A_sz_s(t)+v(t)B_sz_s(t),\quad z_s(0)=z_{s0}\in H_s, \tag{7}$$

where A_u and B_u are respectively the restrictions of A and B in H_u, A_s and B_s are respectively the restrictions of A and B in H_s and z_u and z_s are the components of $z\in H$ on H_u and H_s respectively. On the other hand, since the spectrum of A is discrete, then we can chose η such that $\{\lambda_i/\lambda_i>-\eta\}=\{0\}$ and hence $A_u=0$. In other words, the kernel of A is H_u. Thus the approach consists on splitting the system into the kernel of A and its supplementary H_s.