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Original Article

Optimization of the parameters of surfaces by interpolating variational bicubic splines☆

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Abstract

In this paper we present an interpolation method from a surface or a data set by the optimization of a quadratic functional in a bicubic splines functional space. The existence and the uniqueness of the solution of this problem are shown and as well a convergence result of the method is established. The mentioned functional involves some real non negative parameters; the optimal surface is obtained by a suitable optimization of these parameters. Finally, we analyze some numerical and graphic examples in order to prove the efficiency of the presented method.

AMS: 65D05; 65D07; 65D10; 65D17

Keywords: Optimization; Variational method; Interpolation; Bicubic; Splines

1. Introduction

Several variational methods using the minimization approach of a given functional in an adequate subspace of Sobolev are commonly encountered problems. Such kind of methods have many applications in CAD, CAGD and Earth Sciences. In Geology and Structural Geology the reconstruction of free form surfaces from a Lagrange or Hermite data set have received a considerable attention. Several works have used such mentioned variational approach by minimizing some specific functional (see for example [1,7,8,12]). Such functional can represent for example minimal energy [3], or physical considerations: such as minimization air of surface, curvature or variation of curvature (see [6]). In the same literature one can consult the paper [9], meanwhile in the recent published paper [10] the authors have applied a domain decomposition method for solving large bivariate scattered data fitting problems with three methods based on bivariate splines, namely, the minimal energy method, the discrete least-squares method and the penalized least-squares method. They have studied separately these three methods. Almost always in these articles mentioned above, the minimizing terms are controlled by a corresponding parameter vector. The estimation of the values of these

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parameters that minimize a given measure is an important problem that has been studied in Refs. [5,14,15], for noisy data.

We propose in this work a variational method of the optimization of the parameters specifically minimizing a suitable functional from a Lagrange data set in a bicubic functional space. We study an optimization interpolating method of surfaces in order to obtain a pleasing shape. By minimizing a quadratic functional that contains some terms associated with Sobolev semi-norms we obtain a new function that we call interpolating variational bicubic spline. We study some characterization of this function and we shall compute it in a space of bicubic splines. Moreover, under adequate hypotheses we prove that such bicubic spline converges to a given function from which are proceeding the data.

The primary motivation of this paper is the following: Sobolev semi-norms in the minimizing quadratic functional are controlled by some parameters, in all our work [6–8] we have been asked if we have done any research on how to obtain an optimal parameter. So, the goal of this paper is the study of the estimation of the values of these parameters, and establishes a method to obtain an optimal parameter in order to obtain an optimal interpolated surface with exact data instead of noisy data, which shows that the study of this work is totally different from those of Refs. [5,14,15]. The validation and the effectiveness of our method are shown by presenting some numerical and graphic examples.

The remainder of this paper is organized as follows. In Section 2, we briefly recall some preliminary notations and results. Section 3 is devoted to state an interpolation problem and to define and characterize the interpolating variational bicubic spline. In Section 4, we study how to compute such bicubic spline in practice, while a result about convergence is carefully established in Section 5. In Section 6, we present a parameter optimization method. Finally, Section 7 is dedicated to illustrate some numerical and graphic examples.

2. Notations and preliminaries

For each $k \in \mathbb{N}$, we denote by $\langle \cdot \rangle_k$ and $\langle \cdot, \cdot \rangle_k$, respectively, the Euclidean norm and inner product in $\mathbb{R}^k$. Given $a, b, c, d \in \mathbb{R}$ with $a<b$ and $c<d$, we consider the intervals (a, b), (c, d) and the rectangle $R=(a, b)\times(c, d)$. Given $\Omega \subset \mathbb{R}^k$, we denote by $\mathbb{P}_n(\Omega)$ the linear space of the real polynomials with total degree less than or equal to n defined from Ω into $\mathbb{R}^k$.

Let $H^3(R)$ be the usual Sobolev space of (classes of) functions u belonging to $L^2(R)$, together with all their partial derivatives $D^\beta u$ with $\beta=(\beta_1, \beta_2, \beta_3)^T$, in the distribution sense, of order $|\beta|=\beta_1+\beta_2+\beta_3 \leq 3$. This space is equipped with the norm

$$\|u\| = \left(\sum_{|\beta|\leq 3} \int_R (D^\beta u(x))^2 dx \right)^{1/2},$$

the semi–norms

$$|u|_\ell = \left(\sum_{|\beta|=\ell} \int_R (D^\beta u(x))^2 dx \right)^{1/2}, 0 \leq \ell \leq 3,$$

and the corresponding inner semi-products

$$(u, v)_\ell = \sum_{|\beta|=\ell} \int_R D^\beta u(x) D^\beta v(x) dx, \quad 0 \leq \ell \leq 3.$$

For any $n, m \in \mathbb{N}^*$ let $T_n=\{x_0, \ldots, x_n\}$ (resp. $T_m=\{y_0, \ldots, y_m\}$) be a subset of distinct points of $[a, b]$ (resp. of $[c, d]$), with $a=x_0<x_1<\cdots<x_{n-1}<x_n=b$ (resp. $c=y_0<y_1<\cdots<y_{m-1}<y_m=d$). We denote by $S_3(T_n)$ and $S_3(T_m)$ the spaces of cubic spline functions given by

$$S_3(T_n) = \left\{ s \in C^2(a, b) | s|_{[x_{i-1}, x_i]} \in \mathbb{P}_3[x_{i-1}, x_i], \quad i = 1, \ldots, n \right\},$$

$$S_3(T_m) = \{ s \in C^2(c, d) | s|_{[y_{i-1}, y_i]} \in \mathbb{P}_3[y_{i-1}, y_i], \quad i = 1, \ldots, m \}.$$