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Original article

On the chaotic behaviour of a simple dry-friction oscillator

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Abstract

In this paper we investigate an early and yet simple model used for the analysis of mechanical systems incorporating Coulomb-type friction. We show an interesting non-smooth bifurcation of the crossing-sliding type that causes symmetry breaking. In its simplicity it was not obvious for a long time to find chaos in the simple one degree-of-freedom sliding block model with dry-friction. With the introduction of static coefficient of friction besides the dynamic one we found chaotic bands over a wide range of parameters. In this work we also highlight the possibility of transient chaos for a narrow range of parameters.

Keywords: Dry-friction; Piecewise-smooth system; Lyapunov exponent

1. Introduction

Filippov-type non-smooth systems like those involving dry-friction were extensively studied in the past. Dating back as far as 1930 Den Hartog [5] introduced a very simple mechanical model consisting only of a block sliding on a rough surface and supported by a linear spring. The block was periodically forced. He derived the exact solution for symmetric steady state motions for the case when static and dynamic coefficients of friction had the same value. More than five decades later Shaw [13] highlighted the possibility of introducing a static coefficient of friction different from the dynamic one. He carried out a detailed stability and bifurcation analysis of Den Hartog's original system and found – among others – Hopf bifurcation.

Since then, countless papers have been published that investigate a variety of more complex stick-slip models. These papers address today's engineering problems such as disc brake squeal [7,17] and in many cases they present the possibility of chaos. However, the presence of chaos in Den Hartog's and Shaw's early model did not seem to be verified yet. Some author's interest were drawn to non-smooth bifurcations arising in this simple model and published very detailed analysis reports [4,8,9]. Various types of discontinuity induced bifurcations were revealed for periodic solutions with the help of the so called 'shooting method' and continuation codes like TC-HAT ($\widehat{TC}$) [16].

In this paper we will not focus on the continuation of discontinuity induced bifurcation (DIB) however, this is one of our future aims. We rather concentrate on chaotic behaviour of this simple mechanical system and try to prove it by means of numerical techniques. The paper is organised as follows: in Section 2, we present the mechanical model and the usual non-dimensional form of the equations used in earlier publications concerning the model. We then show the

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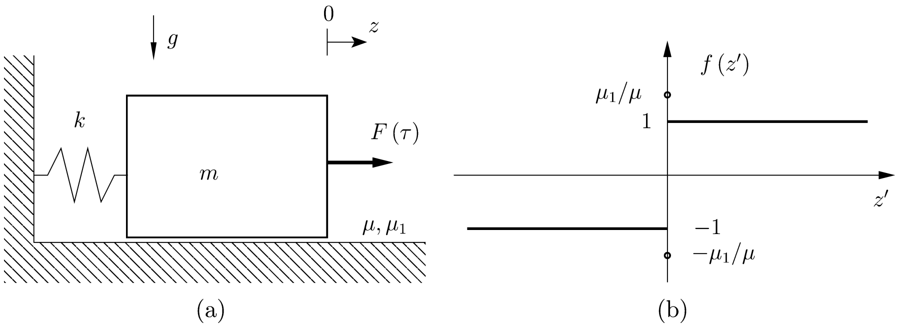

Fig. 1. Panel (a) shows the mechanical model and panel (b) depicts the friction characteristics used.

applied numerical techniques in Section 3. We narrow our analysis to certain parameter ranges where we found chaos. Two methods are presented that were used for the calculation of the maximal Lyapunov exponent. With the help of the results, the possibility of transient chaos will also be highlighted. In Section 4, conclusions will be drawn and future prospects will be outlined.

2. The model

The mechanical model (depicted in panel (a) of Fig. 1) consists of a block of mass m that is supported by a linear spring with stiffness k. The block is sliding on a rough surface so it is subject to friction. The equation of motion is the following in this case:

$$mz'' + kz = F_0 \cos(\omega_0(\tau + \tau_0)) - \mu mgf(z'), \tag{1}$$

where τ is time, $()'$ denotes time derivative, z is the displacement of the block that has mass m, k is the spring stiffness, F_0 is the amplitude of the excitation force, ω_0 is the angular excitation frequency, μ is the sliding coefficient of friction and g is the gravitational acceleration. $\omega_0\tau_0$ is the initial phase angle of the excitation force. Function $f(z')$ can be an arbitrary function of the velocity that models the friction characteristics. There are various friction models that try to handle friction as precisely as possible. Sometimes these models are present in a very sophisticated form and often the complex formulation can lead to chaotic behaviour [1,11]. For simplicity, we chose a piecewise linear function that models the different coefficients for sliding and sticking, otherwise, it is independent of the velocity. It was found in the literature [10,14] that Coulomb-type formulation is sufficient in the case of metallic surfaces. Although there are more complex models like those that consider the so called Stribeck's effect, our main goal was simplicity. The characteristic function can be seen in panel (b) of Fig. 1. In our analysis we use the non-dimensional form of Eq. (1) as written in Eq. (2). We rescaled time by $t = \tau\sqrt{k/m}$ and displacement by $x = zk/F_0$. The non-dimensional constants are $\Omega = \omega_0\sqrt{m/k}$ (non-dimensional excitation frequency), $S = \mu mg/F_0$ (sliding friction force) and $S_1 = \mu_1 mg/F_0$ (sticking friction force). According to this scaling, $f(z') = f((F_0/\sqrt{mk})\dot{x})$. Since $(F_0/\sqrt{mk}) > 0$, $f((F_0/\sqrt{mk})\dot{x}) = f(\dot{x})$. Thus, Eq. (1) can be rewritten as follows:

$$\ddot{x} + x = \cos(\Omega(t + t_0)) - Sf(\dot{x}) \tag{2}$$

$$f(\dot{x}) \in \begin{cases} 1 & \text{if } \dot{x} > 0 \\ [-S_1/S, S_1/S] & \text{if } \dot{x} = 0 \\ -1 & \text{if } \dot{x} < 0 \end{cases},$$

where $\dot{()}$ denotes non-dimensional time derivative.

3. Numerical analysis

Piecewise-smooth dynamical systems can be very challenging when we try to apply analytical calculations. Most of the methods for smooth problems fail essentially. A basic method that can always be applied is the calculation of