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Original article

A strain space framework for numerical hyperplasticity

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Abstract

Numerical simulations of high strain rate plastic flow have historically been built in a hypoelastic framework and use radial return (Wilkins' method) as the solution algorithm. We show how each of these choices can lead to inaccurate and possibly nonconvergent results. We describe an alternative solution procedure based on a simple multiple time scale perturbation theory that is stable, accurate, computationally efficient and simple to implement. Further extension of these results then leads to a strain space formulation that has additional computational advantages. We illustrate our development with numerical experiments.

This paper is dedicated to my friend and colleague Christo Christov on the occasion of his 60th birthday, in recognition of his many important and creative contributions to the formulation of continuum mechanics.

Keywords: Numerical plasticity; Hyperplasticity; Wilkins' method

1. Introduction

In the early days of numerical simulation when computers were slow and memory limited, Mark Wilkins devised an accurate and efficient method for solving the equations of dynamic plasticity. First published in a Livermore technical report [22] in 1963 and the following year in volume 3 of Methods of Computational Physics [21], the method of radial return (often termed Wilkins' method) has been widely cited and is still in common use today. Wilkins' method uses a rate form of the equations in which incremental stress is calculated as a function of stress and rate of deformation; rate formulations in which stress is a history-dependent *functional* are termed hypoelastic [20,3].

At roughly the same time, a theoretical framework for plasticity was being formulated [23,15,17]. Plastic flow is a dissipative process and irreversible thermodynamics is the proper context in which to describe it. The resulting theory, termed thermomechanics, is a hyperelastic theory, which derives from the assumption of a thermodynamic potential. In hyperelastic constitutive models, stress is a *function* of strain and internal variables which contain the history dependence.

An associated theoretical issue was raised by Naghdi and Trapp in 1972 [2,16], namely that plastic yield criteria needed to be formulated in strain space rather than in stress space. Among the issues they raise, at least one is of computational importance, that strain is not necessarily a unique function of stress.

Despite its long term success in the computational mechanics community, Wilkins' method has some shortcomings of both accuracy and stability, especially when applied to high strain rate phenomena. The accuracy issues arise from

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the explicit nature of Wilkins' method and can be readily mitigated by subcycling or implicit solution techniques – possibly at significant computational expense. The stability issues are more subtle, originating in the coupling of radial return with the momentum equation.

In previous work, Margolin and Flower [13] offered an alternative to Wilkins' method for high strain plasticity. The algorithm is based on multiple time scale perturbation theory and addresses issues of both accuracy and stability while preserving computational efficiency. This algorithm is still cast in the framework of hypoelasticity.

There are important advantages to the hyperelastic framework, both theoretical and computational. However, most of the development effort in the computational community has been devoted to improving the constitutive models and in particular to the formulation of yield surfaces in stress space. The implementation of a hyperelastic framework consistent with [16] would require that these models be converted from stress space to strain space and we speculate that this is one of the reasons the computational community does not use hyperelasticity more widely.

The main purpose of this paper is to demonstrate how the perturbation approach of [13] can be extended to convert a stress-based hypoelastic model to a strain-based hyperelastic model for the simple case of a von Mises (constant) yield surface. In Section 2 we briefly review the equations of dynamic elasto-plasticity. In Section 3, we describe Wilkins' method and show that it is a first-order explicit approximation to the analytic equations. In Section 4 we offer numerical experiments to illustrate the potential inaccuracies and to identify the circumstances that are susceptible to error.

In Section 5 we describe a simple multiple time scale theory which leads to an analytic solution of the plasticity equations. The theory is based on two assumptions, namely that the strain rate can be considered approximately constant over a computational cycle, and that the total strain can be additively decomposed into an elastic component and a plastic component. These assumptions are generally valid and not restricted to the case of perfect plasticity.

In Section 6, we modify the multiple time scale theory to replace the yield condition in stress space with a constitutive relation for plastic strain. This represents a first step in creating a true hyperelastic framework that can consistently incorporate modern thermomechanical ideas. We conclude the paper with a discussion in Section 7 of the numerical advantages of our strain space formulation.

2. Elastic–plastic equations

Here, we briefly review the equations that describe time dependent elastic–plastic flow. More detailed discussions and derivations can be found in many classic textbooks, e.g., in Hill [6], Khan and Huang [8], Lubliner [9], etc. Also, many details of the numerical formulation and implementation of plasticity models can be found in Simo and Hughes [19].

Inelasticity in solids is often modeled computationally as plastic flow. In practical terms, finite material strength places limits on the size of the components of the deviatoric stress tensor s_{ij} while dissipating kinetic energy. The total deviatoric strain, $\Delta e_{ij} = \dot{e}_{ij}\Delta t$ realized over some interval of time Δt, will be divided[1] between an elastic or reversible strain Δe^e_{ij}, and a plastic or irreversible strain Δe^p_{ij}:

$$\Delta e_{ij} = \Delta e^e_{ij} + \Delta e^p_{ij}. \tag{1}$$

The plastic strain causes permanent changes in the reference (i.e., unloaded) state of the material. The associated plastic work contributes to the material temperature. However, some fraction of the plastic work is consumed in microstructural changes to the material rather than heat [18].

The onset of plastic flow is described by a yield condition that in numerical algorithms is usually formulated in stress space. Specifically, in rate independent (*perfect*) plasticity one specifies a function of the deviatoric stresses, usually the second invariant J_2, to be always less than or equal to the square of the yield strength Y

$$J_2 \equiv \frac{1}{2} s_{jk} s_{jk} \le Y^2. \tag{2}$$

The yield stress is a material property, and may also depend on the pressure, the temperature, the accumulated plastic strain or plastic work, and other internal variables. The yield surface is closed and convex. Conceptually, it divides the

[1] Sometimes a multiplicative decomposition of the deformation gradient is assumed. However, as discussed in [1,4], the additive decomposition is to be preferred for finite strains whereas the two are equivalent for infinitesimal strains we assume.