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Original article

On the performance of high resolution non-oscillating advection schemes in the context of the flow generated by a mixed region in a stratified fluid

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Abstract

The two-dimensional flow generated by a local density perturbation (fully mixed region) in stratified fluid is considered. In order to describe accurately the sharp discontinuity in density at the edge of the mixed region, monotone schemes of high order of approximation are required. Although a great variety of methods have been developed during the last decades, there remains the question of which method is the best. This present paper deals with the numerical treatment of the advective terms in the Navier–Stokes equations in the Oberbeck–Boussinesq approximation. Comparisons are made between the upwind scheme, flux-limiter schemes, namely Minmod, Superbee, van Leer and monotonized centred (MC), monotone adaptive stencil schemes, namely ENO3 and SMIF, and the weighted stencil scheme WENO5. We used the laboratory experimental data of Wu as a benchmark test to compare the performance of the various numerical approaches. We found that the flux limiter schemes have the smallest numerical diffusion. On the other hand, the WENO5 scheme describes the variation of the width of the collapsing region over time most accurately. All considered schemes give realistic patterns of internal gravity waves generated by the collapsing region.

Keywords: Collapse of mixed region; Stratified fluid; High order non-oscillating scheme

1. Introduction

The problem of a two-dimensional fully mixed region collapsing in a continuously density-stratified medium is considered. The interest in this problem stems from the study of a number of geophysical phenomena and a number of technical problems [7,16,18,19,26,21]. For example, because the turbulent wake behind a body traveling through stratified fluid is very slender in the direction of body motion, the flow field and internal waves induced by the wake can be adequately described by studying the collapse of a non-turbulent mixed region in stratified medium [9,16,20,29,30]. There are numerous studies focusing on locally homogeneous perturbations of the density field (fully mixed region). Experimental work on this problem has been done by Wu [33]. To the best of the authors' knowledge, Wessel [32] was the first who solved the Navier–Stokes equations in the Oberbeck–Boussinesq approximation numerically for the case

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of sharp discontinuity in density at the mixed region edge. He obtained general corroboration with the experiment of Wu [33] in terms of wave patterns and the horizontal size of the mixed region for large time instants. The properties of the internal wave patterns are well described in Lytkin and Chernykh [17] for the laminar collapse of a mixed region having various initial density perturbations in a linearly stratified medium. This problem was used by several researchers as a benchmark test in order to assess the performance of some numerical algorithms which they constructed [2,10]. In Nartov and Chernykh [22] to study the history of the shape of the mixed region, the idea of a non-diffusing passive scalar was used. The method for the localization of singularities [31] was utilized to define the location of the edge of the mixed region. A more complete list of references and more comprehensive overview of research can be found in Chernykh and Voropaeva [5].

Incompressible viscous flow at variable density presents the difficulty of satisfying the property of mass conservation in two respects. On the one hand, the mass density of each fluid particle must remain unchanged during the fluid motion, whatever the level of unsteadiness and mixing. On the other hand, the velocity field must satisfy the incompressibility constraint which reflects the inability of pressure to do compression work. To describe the sharp discontinuity in density at the mixed region edge with sufficient accuracy, schemes of high order of approximation are required. Godunov's theorem [6] states that any linear monotonic advection schemes cannot provide better than first-order accuracy. Therefore, there is a need to apply higher order accuracy numerical schemes derived for numerical solutions of conservation laws which support discontinuous solutions. Many methods match additional requirements, for example; they are Lax–Wendroff, Lax–Friedrichs, flux corrected transport (FCT) methods of Boris–Book and Zalesak, slope limiter methods of van Leer, essentially non-oscillatory (ENO) schemes of Harten–Shu–Osher and total variation diminishing schemes (TVD). Even though there are very little theoretical results about the properties of such schemes in multidimensional and nonlinear cases, in practice these schemes are very robust and stable, and they are used in a lot of practical applications. Yet, there is always the question of what is the best choice, the answer to which is obviously problem dependent. In this research several high order resolution advection schemes will be used to solve the problem of mixed region dynamics in a stratified fluid. A comparative analysis of the effectiveness of the advection schemes will be obtained.

2. Mathematical formulation

The governing equations are the Navier–Stokes equations in the Oberbeck–Boussinesq approximation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{1}{\rho_0}\frac{\partial p_1}{\partial x} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right), \quad (1)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{1}{\rho_0}\frac{\partial p_1}{\partial z} = \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) - g\frac{\rho_1}{\rho_0}, \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (3)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + w\frac{\partial \rho}{\partial z} = 0. \quad (4)$$

Here ν is the coefficient of kinematic viscosity, u and w are the components of velocity in x and z directions of Cartesian coordinates as shown in Fig. 1, $\rho = \rho(x, z, t)$ is the density, $\rho_1(x, z, t) = \rho(x, z, t) - \rho_s(z)$, $\rho_s = \rho_s(z)$ is the density of undisturbed media, $\rho_0 = \rho_s(0)$, p_1 is the deviation of pressure from hydrostatic pressure, and g is the gravitational acceleration. The stratification is assumed to be linear and stable, i.e., $d\rho_s/dz = -a\rho_0$, where $a = const > 0$. The boundary and initial conditions are

$$\rho_1 = 0, \quad u = w = 0, \quad \text{if} \quad x^2 + z^2 \to \infty, \quad t \geq 0, \quad (5)$$

$$\rho = \begin{cases} \rho_0, & \text{if} \quad (x, z) \in A, \quad t = 0, \\ \rho_s(z), & \text{if} \quad (x, z) \notin A, \quad t = 0, \end{cases} \quad (6)$$

$$u = w = 0, \quad \text{if} \quad -\infty < x, z < \infty, \quad t = 0. \quad (7)$$

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