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Internal scales and dispersive properties of microstructured materials

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Abstract

The Mindlin–Engelbrecht–Pastrone model is used for describing 1D longitudinal waves in microstructured solids. The effect of the underlying microstructure is best seen in the emergence of the optical dispersion branch. Dispersive properties of the Mindlin–Engelbrecht–Pastrone model are analyzed. It is shown by making use of the solutions to the boundary value problem that the influence of the optical dispersion branch has a significant effect on wave motion as shown in numerical experiments.

Keywords: Dispersion; Microstructure; Boundary value problem

1. Introduction

Modern science has always been interested in the internal structure of matter. In contemporary material science, leaving aside nanomaterials, the microstructure of most materials is not on the scale of atoms, but rather is on the order of micrometers. Such a microstructure is used not only for explaining various properties of the materials but also used for the engineering of solids as in case of functionally graded materials or composites.

Classical theories of continua aim to smooth out the inhomogeneities created by the underlying microstructure. Contemporary technology is, however, characterized by high-frequency and high-intense impacts which must be included in modeling wave motion. Indeed, in this case, the wavelengths can be comparable to the internal scale of the microstructure and the internal structure of a material becomes important in engineering applications. Consequently, there is an urgent need to understand how wave processes are influenced by underlying microstructure.

The effect of the underlying microstructure on wave propagation is best seen in dispersion, where the velocity of the wave is not constant but depends on the wavenumber of the harmonics. This is due to the length scale that the internal structure brings into the models. If the wavelength of the wave is much larger than the internal scale, the waves do not 'feel' the internal structure and the classical theories give good predictions. When the wavelengths approach the internal scale, the wave propagation is strongly influenced by the underlying microstructure [9].

Furthermore, in the case of the microstructure, dispersion is not only characterized by the variation in the velocity of the wave but also in the emergence of high frequency, so-called 'optical dispersion branches' which reflect the additional degrees of freedom (i.e., deformation and rotation of the microstructure). The presence of these optical

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branches adds more modes of wave propagation into the model, hence making the wave propagation considerably more complex. The classical theories of homogeneous continua fail to predict the existence of the optical dispersion branches [9].

Although dispersion analysis clearly shows that the optical dispersion branches emerge due to the microstructure, it cannot provide insight into the question whether the optical dispersion branches create significant effects when wave propagation is considered. Moreover, as the optical dispersion branches are high frequencies, then these oscillations are often discarded as numerical errors. This is why the numerical experiments must be accompanied by the dispersion analysis.

Over the previous decades many theories of microstructured solids have been proposed which can roughly be divided into two groups: models based on lattice theories [1,2,12,14] and continuum theory [6,8,11,15]. Here we follow the ideas of Mindlin who modeled the microstructure 'as a molecule of a polymer, a crystallite of a polycrystal, or a grain of a granular material' [15]. One-dimensional version of the Mindlin model derived by Engelbrecht et al. [6,7] will serve as the basis for our analysis. Based on the separation of macro- and microstructure of a material, this model, called the Mindlin–Engelbrecht–Pastrone model, is characterized by a clear physical structure of the governing equation.

The paper is organized as follows: the basic model following [6,7] is presented in Section 2. In Section 3, dispersion analysis of the Mindlin–Engelbrecht–Pastrone is carried out and in Section 4, the boundary value problem is introduced. The results and discussion can be found in Section 5. Conclusions are given in Section 6.

2. Basic model

The basic model follows Mindlin [15], as formulated by Engelbrecht and Pastrone [6,7].

The main idea is to distinguish between macro- and micro-displacements $u_i(x_i, t)$ and $u'_j(x'_i, t)$, respectively. Assuming that the micro-displacement is defined in coordinates x'_k moving with a microvolume, we define $u'_j = x'_k\varphi_{kj}(x_i, t)$, where φ_{kj} is an arbitrary function. Because $\partial u'_j/\partial u'_i = \partial'_i u'_j = \varphi_{ij}$, this arbitrary function gives micro-deformation. Since we will consider the simplest 1D case, we will drop the indices i and j.

The fundamental balance laws can be formulated separately for macroscopic and microscopic scales. Introducing the Lagrangian $L = K - W$, formed from the kinetic and potential energies

$$K = \frac{1}{2}\rho u_t^2 + \frac{1}{2}I\varphi_t^2, \quad W = W(u_x, \varphi, \varphi_x), \tag{1}$$

where ρ and I denote the macroscopic density and the microinertia, respectively. The indices x and t denote differentiation.

Using Euler-Lagrange equations and recognizing that

$$\sigma = \frac{\partial W}{\partial u_x}, \quad \eta = \frac{\partial W}{\partial \varphi_x}, \quad \tau = \frac{\partial W}{\partial \varphi}, \tag{2}$$

we obtain the equations of motion

$$\rho u_{tt} = \sigma_x, \quad I\varphi_{tt} = \eta_x - \tau, \tag{3}$$

where σ is the macro- and η is the microstress and τ is the interacting force.

The simplest potential energy function describing the influence of the microstructure is a quadratic function

$$W = \frac{1}{2}\alpha u_x^2 + Au_x\varphi + \frac{1}{2}B\varphi^2 + \frac{1}{2}C\varphi_x^2 \tag{4}$$

where α, A, B and C are constants and depend upon the material. The physical meanings of these parameters are related to bulk (α) and microstress modulus (C), coupling effects (A) and the interactive force (B). The stresses (2) then take the forms

$$\sigma = \alpha u_x + A\varphi, \quad \eta = C\varphi_x, \quad \tau = Au_x + B\varphi, \tag{5}$$

and the equations of motion (3) become

$$\rho u_{tt} = \alpha u_{xx} + A\varphi_x, \quad I\varphi_{tt} = C\varphi_{xx} - Au_x - B\varphi. \tag{6}$$