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Original Article

Compound waves in a higher order nonlinear model of thermoviscous fluids

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Abstract

A generalized traveling wave ansatz is used to investigate compound shock waves in a higher order nonlinear model of a thermoviscous fluid. The fluid velocity potential is written as a traveling wave plus a linear function of space and time. The latter offers the possibility of predicting the outcome of interacting shock waves, i.e. shock jump heights and wave velocities after collisions and overtakes. The stability of the linear solution part is investigated and a criterion for its stability is determined. For a number of instances, the numerical results show formation of rarefaction waves. By using a similarity transformation, analytical expressions for these rarefaction waves are found in the limit of no dissipation. Examples of compound shock waves are illustrated by numerical simulations.

Keywords: Shock waves; Traveling fronts; Thermoviscous fluids; Shock interactions

1. Introduction

Shock waves appearing in thermoviscous fluids are solitary waves resulting from balancing nonlinearity with viscous and heat conducting effects. The traveling wave approach has predominantly been used for nonlinear partial differential equations of Hamiltonian type and for reaction diffusion problems. However, it is well known that the traveling wave ansatz can be used to find shock waves in Burgers' equation. Despite this fact it has only recently been appreciated that the solitary wave approach is well suited for studies of various models of thermoviscous shocks. Jordan [12] determined a traveling wave solution for the Kuznetsov equation [16] and later on successfully invoked the traveling

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wave approach for studies of nonlinear viscoelastic media [13,14]. In order to investigate high Mach number shock wave propagation, Chen et al. [3] investigated a higher-order equation derived by Söderholm [24], allowing for a more accurate assessment of traveling wave velocities.

In this paper compound shock waves are investigated in the model for thermoviscous fluids proposed by Söderholm [24]. The studies are based on a generalization of the traveling wave ansatz for the velocity potential by adding a function linear in the space and time variables to the traveling wave part [21,23]. The solution of the resulting ordinary differential equation is given implicitly, in contrast to the explicit solutions found for the Kuznetsov equation in Ref. [12] and a third order approximation to acoustic waves in thermoviscous fluid in Refs. [21,23]. The generalized ansatz makes it possible to determine analytically the outcome of head-on colliding and overtaking shock waves. In order to illustrate the solitary wave properties (or quasi soliton nature) of the traveling shock waves, collision and overtake simulation experiments are performed for the shocks. The traveling wave shock solutions are equivalent to the Taylor shock solution of the Burgers equation. However, in contrast to the Burgers equation the model studied here allows counter propagating shocks.

The paper is organized as follows: In Section 2 the model equation is presented, in Section 3 a generalized traveling wave ansatz is used to determine an implicit shock wave solution and in Section 4 rarefaction waves are investigated. In Section 5 stability properties are studied and finally Section 6 deals with compound waves.

2. Model equation

In this investigation we use a model derived by Söderholm [24]. The wave propagation phenomena are restricted to the case of plane waves with finite amplitudes in one spatial dimension and in a homogeneous medium. The fluid particle velocity field is denoted by $u = u(x, t)$, where x is the space variable and t is time. The wave equation is formulated in terms of the velocity potential $\psi = \psi(x, t)$ defined by

$$u \equiv -\psi_x, \tag{1}$$

where subscript denotes partial differentiation. The dynamical equation governing the acoustic wave propagation reads in terms of ψ [24]

$$\psi_{tt} - c_0^2\psi_{xx} = (\gamma - 1)\psi_{xx}\psi_t + 2\psi_{xt}\psi_x - \frac{\gamma + 1}{2}(\psi_x)^2\psi_{xx} + b\psi_{xxt}. \tag{2}$$

Here γ is the adiabatic index or ratio of the specific heats, c_0 is the small-signal speed of sound, and b is the diffusivity of sound [10], which takes into account thermal and viscous losses. Eq. (2) is the one-dimensional version of the three-dimensional model equation derived by Söderholm [24], taking only first order dissipative effects into account. The derivation of the model in (2) is based on conservation of mass, i.e. the continuity equation, the momentum equation including shear and bulk viscosity, the entropy equation for heat transfer and finally an equation of state relating pressure p to the fluid density ρ and its entropy s [24,22]. In the model nonlinear contributions to dissipation is neglected. In this case entropy can be eliminated reducing the number of equations. A key point is the equation of state $p(\rho, s)$ giving the pressure dependence on density ρ and entropy s. The parameter γ in Eq. (2) originates from the equation of state, and the dissipation parameter b depends on the specific heats. In order to discuss this we consider the first order approximation for the pressure

$$p(x, t) \simeq p_0 + \rho_0\eta(x, t) \tag{3}$$

where p_0 (ρ_0) is the static pressure (density) and $\eta \equiv \psi_t$. The lossless limit of Eq. (2), which is obtained by letting $b = 0$, appears in a number of works [10,20,2,19,9]. Some of these authors emphasize the fact that the equation is exact for a lossless perfect gas, which is described by the following equation of state

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma}, \quad c^2 = \frac{dp}{d\rho} = \gamma\frac{p}{\rho}, \quad c_0^2 = \gamma\frac{p_0}{\rho_0}, \tag{4}$$

where ρ is the density and c is the velocity of sound. For $b = 0$ Eq. (2) is exact in the sense that it can be derived from the Euler equations without introducing any approximations. Accordingly, Christov et al. [5,4,6] denoted Eq. (2) with $b = 0$ the potential Euler equation.