

## Original articles

# Construction of nonstandard finite difference schemes for the SI and SIR epidemic models of fractional order

Abraham J. Arenas<sup>a</sup>, Gilberto González-Parra<sup>b,c,\*</sup>, Benito M. Chen-Charpentier<sup>c</sup><sup>a</sup> *Departamento de Matemáticas y Estadística, Universidad de Córdoba, Montería, Colombia*<sup>b</sup> *Grupo de Matemática Multidisciplinar (GMM), Fac. de Ingeniería, Universidad de los Andes, Mérida, Venezuela*<sup>c</sup> *Department of Mathematics, University of Texas at Arlington, Arlington, TX 76019-0408, United States*

Received 19 August 2014; received in revised form 13 August 2015; accepted 3 September 2015

Available online 25 September 2015

## Abstract

In this paper we construct nonstandard finite difference (NSFD) schemes to obtain numerical solutions of the susceptible–infected (SI) and susceptible–infected–recovered (SIR) fractional-order epidemic models. In order to deal with fractional derivatives we apply the Caputo operator and use the Grünwald–Letnikov method to approximate the fractional derivatives in the numerical simulations. According to the principles of dynamic consistency we construct NSFD schemes to numerically integrate the fractional-order epidemic models. These NSFD schemes preserve the positivity that other classical methods do not guarantee. Additionally, the NSFD schemes hold other conservation properties of the solution corresponding to the continuous epidemic model. We run some numerical comparisons with classical methods to test the behavior of the NSFD schemes using the short memory principle. We conclude that the NSFD schemes, which are explicit and computationally inexpensive, are reliable methods to obtain realistic positive numerical solutions of the SI and SIR fractional-order epidemic models.

© 2015 International Association for Mathematics and Computers in Simulation (IMACS). Published by Elsevier B.V. All rights reserved.

**Keywords:** Nonstandard finite difference methods; Positive solutions; Epidemic models; Derivative of fractional order

## 1. Introduction

Health issues are some of the most important problems that need to be dealt in the world and, particularly, epidemics have received a lot of attention. The dynamics of epidemic diseases have been studied for a long time. There are several approaches to study the dynamics of epidemics. One important branch uses models that include time derivatives and gives systems of ordinary differential equations. In these models each equation represents the change in the number of individuals in various categories given by continuous variables. The most widely discussed type of infection spread is the SIR system, in which individuals are susceptible (S), infective (I) or removed/immune (R) [28,43]. There exist

\* Corresponding author at: Grupo de Matemática Multidisciplinar (GMM), Fac. de Ingeniería, Universidad de los Andes, Mérida, Venezuela.

E-mail addresses: [aarenas@correo.unicordoba.edu.co](mailto:aarenas@correo.unicordoba.edu.co) (A.J. Arenas), [gcarlos@ula.ve](mailto:gcarlos@ula.ve) (G. González-Parra), [bmchen@uta.edu](mailto:bmchen@uta.edu) (B.M. Chen-Charpentier).

other classical epidemic models such as the SI, SIS, SIR, SIRS and SEIR. Additionally, several other types of models have been created in order to deal with epidemics.

Fractional order models have been incorporated in several areas of science, engineering, applied mathematics, economics, and bioengineering and consequently considerable attention has been given to the solutions of fractional differential equations [22,1,29,4,32]. Fractional derivatives have the unique property of capturing the history of the variable, that is, they have memory. Also the effect of recent memory is more important than the effect of older memory [11]. This cannot be easily done by means of the integer order derivatives [48,19,13,10]. The application of fractional derivative models to several problems is also justified since they provide a better agreement with real data than integer order derivative models. The value of the index of the fractional derivative,  $\alpha$ , characterizes the way in which the memory along different parts of the interval of integration affects the solution at a given time and it can be varied to best fit the real data. This is the case for many problems in biology, physics and engineering [48,53,19,13,10,17]. A specific example is [19] where the authors proposed a fractional order model for HIV based on the fact that the memory is one of the main features of immune response.

Recently, fractional derivative models have been developed for some epidemics [48,52,25,53,54,9,47,21,27]. A Malaria fractional order has been proposed in [48] with the aim to build a better approximation model to the real dynamics of malaria transmission among heterogeneous populations. Moreover, in [17] a mathematical model of fractional order for the simulation of the dynamics of a dengue fever outbreak is proposed. The author obtained a much better approximation and additionally concluded that the behavior of the human population follows a model of a different order than the mosquito population. Also, in [53] the authors noted that the classical integer classical first-order SEIR model was unable to reproduce the statistical data collected in a real outbreak of Dengue with a sufficient degree of accuracy. A similar conclusion regarding AH1N1/09 influenza was given in [23]. In [3] the authors studied a fractional order model for healthy and infected foxes with rabies.

It is important to remark that fractional differential operators have a long history, having been mentioned by Leibniz in a letter to l'Hospital in 1695, [55]. Following l'Hospital's and Leibniz's first inquisition, fractional calculus was primarily a study reserved to the best minds in mathematics. Fourier, Euler, and Laplace are among the many that contributed to the development of fractional calculus. Several analytical and numerical techniques have been developed to solve fractional differential equations [22,56,51,34,5,55]. However, many of the analytical techniques are applicable only to a limited class of fractional differential equations, and the closed form solutions are expressed in terms of infinite series which are difficult to handle. One of the major drawbacks of the numerical techniques is that they require storing of the past response, and if the simulation time is large, the memory required to store the past data and the computational cost can also be large [2].

The numerical methods in many cases generate numerical instabilities i.e. numerical solutions that do not correspond to any of the solutions of the original system of differential equations. The most common numerical instabilities involve the introduction of dynamical features, such as period-doubling bifurcations, chaos, divergence of solutions, etc., which are not consistent with the dynamics of the original continuous time model being discretized [37,31,20]. In regard to fractional differential equations the numerical solutions are more expensive computationally due to the presence of the long and persistent memory, which is related to the nonlocal nature of fractional derivative operators [46,57,15].

On the other hand, in the process of modeling and simulation of many phenomena, it is important that the numerical methods guarantee the positivity of the solution. Thus, it is important to construct positivity preserving schemes that avoid unrealistic negative values for the solution [37,12,44,8,26,16,20]. One possible strategy is to construct a nonstandard finite difference (NSFD) schemes that have been introduced in [37] with the aim of avoiding full implicit schemes, which are computationally expensive and may cause unrealistic negative solutions. Additionally, the NSFD schemes are in nature explicit and generally less expensive than other classical methods since larger time steps can be taken without generating negative solutions [37,39,16]. The NSFD schemes, also preserve some of the main essential physical properties of the solution, such as, monotonicity or convergence towards a stable steady-state [37,46,8,26]. It is important to remark that NSFD methodology was developed by Mickens and has been applied in many areas of science including biology and epidemic models [37,41,24]. More recent works include applications for models based on partial differential equations, economic models and diseases [35,12,20,26,54]. Moreover, a first nonstandard numerical scheme has been developed for the fractional-order Brusselator system [46]. Additionally, a mathematical model based on fractional differential equations has been developed for Psoriasis and numerically solved by means of a nonstandard difference scheme [54].

Download English Version:

<https://daneshyari.com/en/article/1139185>

Download Persian Version:

<https://daneshyari.com/article/1139185>

[Daneshyari.com](https://daneshyari.com)