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Original articles

Asymptotic behavior of Manakov solitons: Effects of potential wells and humps

V.S. Gerdjikov[a], M.D. Todorov[b,*], A.V. Kyuldjiev[a]

[a] *Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 72 Tzarigradsko Chaussee, Blvd., 1784 Sofia, Bulgaria*
[b] *Department of Applied Mathematics and Computer Science, Technical University of Sofia, 8 Kliment Ohridski, Blvd., 1000 Sofia, Bulgaria*

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Highlights

- The asymptotics of soliton solutions of the perturbed Manakov system is considered.
- Three-soliton chains perturbed by sech-like external potentials are studied.
- The Manakov system and the perturbed complex Toda chain are compared.
- Perturbed complex Toda chain predicts the long-time evolution of the Manakov system.

Abstract

We consider the asymptotic behavior of the soliton solutions of Manakov's system perturbed by external potentials. It has already been established that its multisoliton interactions in the adiabatic approximation can be modeled by the complex Toda chain (CTC). The fact that the CTC is a completely integrable system, enables us to determine the asymptotic behavior of the multisoliton trains. In the present study we accent on the 3-soliton initial configurations perturbed by sech-like external potentials and compare the numerical predictions of the Manakov system and the perturbed CTC in different regimes. The results of conducted analysis show that the perturbed CTC can reliably predict the long-time evolution of the Manakov system.
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Keywords: Perturbed vector Schrödinger equation; Manakov system; Generalized complex Toda chain; Multisoliton interaction in adiabatic approximation

1. Introduction

The Gross–Pitaevski (GP) equation and its multicomponent generalizations are important tools for analyzing and studying the dynamics of the Bose–Einstein condensates (BEC), see the monographs [37,22,26] and the numerous references therein among which we mention [33,4,3,24,36,43,44,16,31,23,8,27,29]. In the 3-dimensional case these equations can be analyzed solely by numerical methods. If we assume that BEC is quasi-one-dimensional then the GP

* Corresponding author.
E-mail addresses: gerjikov@inrne.bas.bg (V.S. Gerdjikov), mtod@tu-sofia.bg (M.D. Todorov), kyuljiev@inrne.bas.bg (A.V. Kyuldjiev).

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equations mentioned above may be reduced to the nonlinear Schrödinger equation (NLSE) perturbed by the external potential $V(x)$

$$iu_t + \frac{1}{2}u_{xx} + |u|^2 u(x,t) = V(x)u(x,t), \tag{1}$$

or to its vector generalizations (VNLSE)

$$i\vec{u}_t + \frac{1}{2}\vec{u}_{xx} + (\vec{u}^\dagger, \vec{u})\vec{u}(x,t) = V(x)\vec{u}(x,t). \tag{2}$$

The Manakov model (MM) [32] is a two-component VNLSE with $V(x) = 0$ (for more details see [1,20]).

The analytical approach to the N-soliton interactions was proposed by Zakharov and Shabat [46,35] for the scalar NLSE, for a vector NLSE see [28]. They treated the case of the exact N-soliton solution where all solitons had different velocities. They calculated the asymptotics of the N-soliton solution for $t \to \pm\infty$ and proved that both asymptotics are sums of N one-soliton solutions with the same sets of amplitudes and velocities. The effects of the interaction were shifts in the relative center of masses and phases of the solitons. The same approach, however, is not applicable to the MM, because the asymptotics of the soliton solution for $t \to \pm\infty$ do not commute.

The present paper is an extension of [10,17] where the main result is that the N-soliton interactions in the adiabatic approximation for the Manakov model can also be modeled by the CTC [14,19,13,18]. More specifically, here we continue our analysis of the effects of external potentials on the soliton interactions. While in [10,17,8] we studied the effects of periodic, harmonic and anharmonic potentials, here we consider potential wells and humps of the form:

$$V(x) = \sum_s c_s V_s(x, y_s), \qquad V_s(x, y_s) = \frac{1}{\cosh^2(2\nu_0 x - y_s)}. \tag{3}$$

If c_s is negative (resp. positive) $V_s(x)$ is a well (resp. hump) with width 1.7 at half-height/depth. Adjusting one or more terms in (3) with different c_s and y_s we can describe wells and/or humps with different widths/depths and positions.

In the present paper we in fact prove the hypothesis in [6] and extend the results in [6,8,9,30,38,45,10,7,15] concerning the model of soliton interactions of vector NLSE (2) in adiabatic approximation.

The corresponding vector N-soliton train is a solution of (2) determined by the initial condition:

$$\vec{u}(x, t=0) = \sum_{k=1}^{N} \vec{u}_k(x, t=0), \qquad \vec{u}_k(x,t) = u_k(x,t)\vec{n}_k, \qquad u_k(x,t) = \frac{2\nu_k e^{i\phi_k}}{\cosh(z_k)} \tag{4}$$

with

$$\begin{aligned} z_k &= 2\nu_k(x - \xi_k(t)), \qquad \xi_k(t) = 2\mu_k t + \xi_{k,0}, \\ \phi_k &= \frac{\mu_k}{\nu_k} z_k + \delta_k(t), \qquad \delta_k(t) = 2(\mu_k^2 + \nu_k^2)t + \delta_{k,0}, \end{aligned} \tag{5}$$

where the s-component polarization vector $\vec{n}_k = \left(n_{k,1}e^{i\beta_{k,1}}, n_{k,2}e^{i\beta_{k,2}}, \ldots, n_{k,s}e^{i\beta_{k,s}}\right)^T$ is normalized by the conditions

$$\langle \vec{n}_k^\dagger, \vec{n}_k \rangle \equiv \sum_{p=1}^{s} n_{k,p}^2 = 1, \qquad \sum_{p=1}^{s} \beta_{k;s} = 0. \tag{6}$$

The adiabatic approximation holds true if the soliton parameters satisfy [25]:

$$|\nu_k - \nu_0| \ll \nu_0, \qquad |\mu_k - \mu_0| \ll \mu_0, \qquad |\nu_k - \nu_0|\, |\xi_{k+1,0} - \xi_{k,0}| \gg 1, \tag{7}$$

for all k, where $\nu_0 = \frac{1}{N}\sum_{k=1}^{N} \nu_k$, and $\mu_0 = \frac{1}{N}\sum_{k=1}^{N} \mu_k$ are the average amplitude and velocity, respectively. In fact we have two different scales:

$$|\nu_k - \nu_0| \simeq \varepsilon_0^{1/2}, \qquad |\mu_k - \mu_0| \simeq \varepsilon_0^{1/2}, \qquad |\xi_{k+1,0} - \xi_{k,0}| \simeq \varepsilon_0^{-1}.$$

We remind that the basic idea of the adiabatic approximation is to derive a dynamical system for the soliton parameters which would describe their interaction. This idea was initiated by Karpman and Solov'ev [25] and

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