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Mathematics and Computers in Simulation 120 (2016) 64–78
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

Statistical inference for the generalized inverted exponential distribution based on upper record values

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Received 29 April 2014; received in revised form 13 April 2015; accepted 29 June 2015
Available online 13 July 2015

Abstract

In this paper, non-Bayesian and Bayesian estimators for the unknown parameters are obtained based on records from the generalized inverted exponential distribution. Bayes' estimators of the unknown parameters are obtained under symmetric and asymmetric loss functions using gamma priors on both the shape and the scale parameters. The Bayes estimators cannot be obtained in explicit forms. So we propose Markov Chain Monte Carlo (MCMC) techniques to generate samples from the posterior distributions and in turn computing the Bayes estimators. We have also derived the Bayes interval of this distribution and discussed both frequentist and the Bayesian prediction intervals of the future record values based on the observed record values. Monte Carlo simulations are performed to compare the performances of the proposed methods, and a data set has been analyzed for illustrative purposes.

Keywords: Bayes estimator; Bayes prediction; General entropy loss function; Maximum likelihood estimator; Median prediction

1. Introduction

The study of record values and associated statistics are of great significance in multiple real life situations such as meteorology, seismology, athletic events, economics, and life testing. The frequency of weather conditions inspired the populace [16] to study distributions of lower records, record times, and inter-record times for independent and identically distributed (iid) sequences of random variables. Since then, numerous papers on record values and their distributional properties have appeared in the statistical literature, among them [2,4,6,9–12,20,24,30,31].

Let $X_{U(1)}, X_{U(2)}, \dots, X_{U(n)}$ be the first n upper record values from the two-parameter generalized inverted exponential distribution (GIED) with probability density function

$$f(x;\alpha,\lambda) = \frac{\alpha\lambda}{x^2} e^{-\lambda/x}(1-e^{-\lambda/x})^{\alpha-1} \quad x \geq 0, \alpha, \lambda > 0 \tag{1}$$

and cumulative distribution function

$$F(x;\alpha,\lambda) = 1-(1-e^{-\lambda/x})^{\alpha} \quad x \geq 0. \tag{2}$$

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http://dx.doi.org/10.1016/j.matcom.2015.06.012

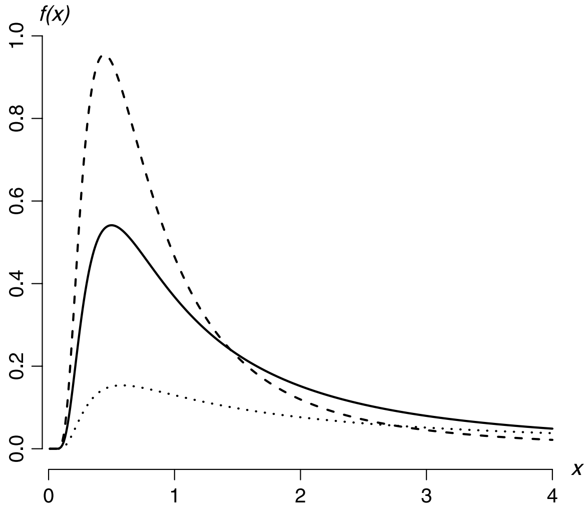

Fig. 1. GIED density functions.

The corresponding hazard function is

$$h(x;\alpha,\lambda) = \frac{\alpha\lambda}{x^2(e^{\lambda/x}-1)} \qquad x \geq 0. \tag{3}$$

Here $\alpha, \lambda > 0$ are the shape and scale parameters, respectively. The generalized inverted exponential distribution with shape parameter α and scale parameter λ will be denoted by GIED(α, λ). The GIED was introduced by [1] as a generalization of the inverted exponential distribution. An extensive studies on the properties of the GIED was carried out by [1]. Due to its practicality, the GIED can be used for several applications, including accelerated life testing, horse racing, supermarket queues, sea currents, wind speeds, and others [29]. The GIED is a special case of the exponentiated Fréchet distribution [29]. [25] studied the reliability estimation of this distribution under progressive type-II censoring. [17] performed several estimation techniques for the unknown parameters from both frequentist and Bayesian perspective utilizing complete sample. Subsequently, [18] obtained the maximum likelihood and the Bayes estimates for the unknown parameters based on hybrid censoring scheme. Recently, [19] also obtained maximum likelihood and Bayes estimates of this distribution on the basis of a progressively type-II censored sample. They also showed the existence, uniqueness and finiteness of the maximum likelihood estimates of the parameters of GIED based on progressively type-II censored data. The hazard function of the GIED can never be constant. The GIED has a unimodal and right skewed density function. In many situations, the GIED provides a better fit than the gamma, Weibull, generalized exponential, and inverted exponential distributions as mentioned in [1]. Fig. 1 shows the probability density functions for the GIED with $\alpha = 1, \lambda = 1$ plotted as the solid line, $\alpha = 2, \lambda = 1$ plotted as the dashed line, and $\alpha = 0.25, \lambda = 1$ plotted as the dotted line. Fig. 2 shows various hazard functions for the GIED using the same parameter settings.

The key role of this article is to present both frequentist and Bayesian methodology for estimating unknown parameters of the GIED based on upper records, and prediction of future observations using current data.

We use maximum likelihood estimation method as a part of frequentist methodology for parameter estimation in Section 2. The asymptotic confidence intervals based on the observed Fisher's information matrix is also taken in account. We consider Bayesian estimation of the unknown parameters in Section 3. The Bayesian inference mainly depends on two features: choice of prior distribution of the parameters and the loss function to be used for Bayesian computations. In this article, we use gamma priors for both scale and shape parameters and they are assumed to be independent of each other. For Bayesian inference, we use a general entropy loss function (GELF). This loss function results in both symmetric and asymmetric loss functions. A brief discussion of this loss function is presented later in Section 3. The joint posterior distribution is not in closed form and thus the posterior sampling is not straightforward to implement. Here we propose a Markov Chain Monte Carlo (MCMC) technique which involve Metropolis–Hastings (M–H) algorithm for posterior sampling. Besides Bayes estimates, we also obtain a two-sided Bayes probability intervals as a Bayesian counter part of the asymptotic confidence intervals in Section 3. Bayes estimates greatly dependent