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Bootstrap prediction in univariate volatility models with leverage effect

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Abstract

The EGARCH and GJR-GARCH models are widely used in modeling volatility when a leverage effect is present in the data. Traditional methods of constructing prediction intervals for time series normally assume that the model parameters are known, and the innovations are normally distributed. When these assumptions are not true, the prediction interval obtained usually has the wrong coverage. In this article, the Pascual, Romo and Ruiz (PRR) algorithm, developed to obtain prediction intervals for GARCH models, is adapted to obtain prediction intervals of returns and volatilities in EGARCH and GJR-GARCH models. These adjustments have the same advantage of the original PRR algorithm, which incorporates a component of uncertainty due to parameter estimation and does not require assumptions about the distribution of the innovations. The adaptations show good performance in Monte Carlo experiments. However, the performance, especially in volatility prediction, can be very poor in the presence of an additive outlier near the forecasting origin. The algorithms are applied to the daily returns series of the GBP/USD exchange rates.

Keywords: Interval prediction; Volatility interval prediction; Interval prediction and outlier; Interval prediction in EGARCH model; Interval prediction in GJR-GARCH model

1. Introduction

The prediction of future values is a key objective in time series analysis, and it is of interest in many areas of knowledge, such as economics, finance, production planning, and sales forecasting. The importance stems from the fact that it is advantageous to know the likely evolution of the series in the future.

Generally, these predictions are given as point estimates, although the prediction interval is even more important [17]. Nevertheless, authors of textbooks on time series analysis and forecasting generally devote little attention to prediction intervals and give little guidance on how to calculate them [5, p. 479]. Also, in general prediction intervals are calculated under the assumption that the model is known and errors are normally distributed.

In the financial time series literature there is little work on procedures to obtain prediction intervals for return and volatility in the GARCH family. Moreover, some stylized facts like (conditional) innovation distribution with heavy

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tail and asymmetry, and the leverage effect affect the coverage of the traditional intervals, making the predictions inadequate and generally leading to a greater risk than is desirable.

An alternative to solve this problem is to obtain prediction intervals using the bootstrap procedure, which does not require any assumption on the distribution of the innovations [7]. In recent decades, several works have proposed bootstrap procedures to construct intervals for time series prediction. In a seminal work in this area, [21] constructed intervals for autoregressive models using bootstrap replicates and fixing the latest p observations, where p is the autoregressive order, [20] also constructed prediction intervals for autoregressive models using bootstrap replicates but start the bootstrap with a randomly chosen block of size p from the observed series. In the financial time series field, we can mention the work of [14] proposing a bootstrap procedure to obtain intervals for forecasting in ARCH processes; [19], who obtained prediction intervals for ARCH models; [18], who extended the procedure presented by [16] to ARIMA models for predicting volatility and return densities in GARCH processes; [6], who proposed new methods for prediction intervals of return and volatility in ARCH and GARCH models; [13], who used bootstrap sub sampling for interval prediction in GARCH models; among others.

One bootstrap method for prediction intervals for volatility models that has shown good results, and that appears to be generalizable to other models is the method proposed by [18] (PRR) for GARCH models. This method incorporates the uncertainty of the estimation in the forecasting interval, since the parameters are estimated at each bootstrap replication. The method also does not depend on the (conditional) innovation distribution. This paper proposes an adaptation of the PRR algorithm developed for prediction intervals for GARCH models, to get prediction intervals for EGARCH and GJR-GARCH models. The paper also studies the effect of additive outliers on the proposed prediction intervals.

The paper is organized as follows: Section 2 introduces the volatility models, Section 3 presents the bootstrap procedures for obtaining prediction intervals for EGARCH and GJR-GARCH models. Section 4 presents the results obtained by simulation; and Section 5 presents an application of the proposed procedures to the daily returns series of the GBP/USD exchange rates. Section 6 concludes.

2. The EGARCH and GJR-GARCH models

GARCH models [3] have been widely used in modeling volatility. Based on this model, other models have been proposed to incorporate other stylized facts, such as the leverage effect. In this sense, we mention the EGARCH [15] and GJR-GARCH models [10]. Given its popularity in empirical applications, in this paper we focus on the EGARCH(1,1) and GJR-GARCH(1,1) models.

Definition 2.1 (*Univariate EGARCH Model*)**.** An EGARCH(1,1) process, $\{r_t\}$, is defined as:

$$\begin{aligned} r_t &= \sigma_t \varepsilon_t, \\ log(\sigma_t^2) &= \omega + \alpha \varepsilon_{t-1} + \gamma(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|) + \beta log(\sigma_{t-1}^2), \end{aligned} \tag{2.1}$$

where $\omega, \alpha, \beta, \gamma$, are real numbers, and $\varepsilon_t \sim IID(0, 1)$ (independent and identically distributed random variables with zero mean and unit variance). Particularly, when $|\beta| < 1$, the EGARCH model is stationary if ε_t comes from Gaussian or Generalized Error Distribution (GED) with shape parameter >1. Nevertheless, if ε_t comes from Student-t or a GED distribution with shape parameter ≤ 1, the EGARCH model is stationary if $\gamma \leq -|\alpha|$. An advantage of this model is that there is no restriction imposed to ensure that the variance is positive.

Definition 2.2 (*GJR-GARCH Model*)**.** A GJR-GARCH(1,1) process, $\{r_t\}$, is defined as:

$$\begin{aligned} r_t &= \sigma_t \varepsilon_t, \\ \sigma_t^2 &= \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma r_{t-1}^2 I(r_{t-1} < 0), \end{aligned} \tag{2.2}$$

where, $I(\cdot)$ is the indicator function, $\varepsilon_t \sim IID(0, 1)$, $\omega > 0$ and α, β, γ are non-negative real numbers for ensuring positive σ_t^2. The model is stationary if $\gamma < 2(1 - \alpha - \beta)$.