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Original article

Global stability and bifurcation of time delayed prey–predator system incorporating prey refuge

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Abstract

This paper describes a prey–predator model with Holling type II functional response incorporating prey refuge. The equilibria of the proposed system are determined and the behavior of the system is investigated around equilibria. Density-dependent mortality rate for the predator is considered as bifurcation parameter to examine the occurrence of Hopf bifurcation in the neighborhood of the co-existing equilibrium point. Discrete-type gestational delay of predators is also incorporated on the system. The dynamics of the delay induced prey–predator system is analyzed. Delay preserving stability and direction of the system is studied. Global stability of the delay preserving system is shown. Finally, some numerical simulations are given to verify the analytical results, and the system is analyzed through graphical illustrations.

MSC: 93C15; 37L10; 93D05

Keywords: Prey–predator; Refuge; Delay; Hopf bifurcation; Global stability

1. Introduction

The dynamic relationship between prey and their predators has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance [4,9,28,31–33]. A prey–predator model with a refuge introduced by one of the founders of population ecology Gause [10,11] and his co-workers to explain discrepancies between their observations and predictions of the Lotka–Volterra [23,30] prey–predator model. They replaced the linear functional response used by Lotka and Volterra by a saturating functional response with a discontinuity at a critical prey density and predicted existence of a limit cycle in predator–prey dynamics. However, Křivan [21] re-analyzed Gause model and shown that the model was ill posed. He defined and analyzed the solutions of the Gause model and concluded three possibilities: (1) trajectories converge to a limit cycle, as predicted by Gause, (2) trajectories converge to equilibrium, or (3) the prey population escapes predator control and grows to infinity. The

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effect of impulse in a stage structured prey–predator ecological model was studied by some authors [27,34]. Again, it may also be noted that several researchers [35,37] considered delay induced prey–predator model to analyze the stability of the system.

Moreover, it is relevant to point that any form of refuge plays an important dynamics to the concerned system. Magalhães et al. [24] carried out a greenhouse experiment on larvae of western flower thrips, *Frankliniella occidentalis* use the web produced by spider mites as a refuge from predation by the predatory mite, *Neoseiulus cucumeris* and developed a stage-structured predator–prey model to incorporate the benefits of refuge use. The model predicted a minor effect of the refuge on the prey density at equilibrium through indicating the effect of refuges on population dynamics. González-Olivares and Ramos-Jiliberto [12] used an analytical approach to study the dynamic consequences of the simplest forms of refuge use by the prey and clarified the role of prey refuges in simple predator–prey models other than the original Lotka–Volterra equations. They concluded that there is a trend of limit cycles through non-zero stable points up to predator extinction and prey stabilizing at high densities.

Kar [18] proposed a prey–predator model incorporating a prey refuge and independent harvesting in either species. He considered harvesting effort as control to prove that it is possible to break the cyclic behavior of the system and drive it to a required state. Ko and Ryu [20] studied a predator–prey model with Holling type II functional response incorporating a prey refuge under homogeneous Neumann boundary condition. They have shown the existence and non-existence of non-constant positive steady-state solutions depending on the constant refuge, which provides a condition towards the protection of prey population from predation. Poggiale et al. [26] studied a general predator–prey system in a spatially heterogeneous environment. They have assumed that the prey have a refuge and shown that the dynamics of the system on a slow time-scale become donor-controlled. In conclusion, they pointed out that in heterogeneous environments, the prey population dynamics depend in a switch-like manner on the presence or absence of predators, not on their actual density.

Chakraborty et al. [5] described a prey–predator fishery model, incorporating prey refuge, reflecting the dynamic interaction between the net economic revenue and the fishing effort used to harvest the prey species in the presence of predation and a suitable tax. They have analyzed the dynamic behavior of the system and shown the occurrence of Hopf bifurcation. Pal and Samanta [25] presented a paper which deals with the problem of a predator–prey model incorporating a prey refuge with disease in the prey–population. They addressed the dynamical behaviours such as boundedness, permanence, local and global stabilities of the system and studied the effect of discrete time delay on the model. They have also estimated the length of delay preserving the stability. Kar [17] considered the prey–predator model with Holling type-II response function incorporation a prey refuge and analyzed the dynamical behavior of the model system.

Chen et al. [6] investigated a predator–prey model with Holling type II functional response incorporating a constant prey refuge. They have shown the instability and global stability properties of the equilibria and the existence and uniqueness of limit cycles for the model. Costa et al. [7] analyzed the dynamics of two predator–prey models, namely, Lotka–Volterra and Leslie–Gower, when submitted to a specific harvesting schedule. Their qualitative analysis proved that the global stability depends on the existence of virtual equilibrium points which in turn is ensured by a suitable choice of the control parameters. Huang [16] considered a prey–predator model with Holling type III response function incorporating a prey refuge. They have analyzed the model and discussed some significant qualitative results from biological point of view. Cressmana and Garay [8] developed a refuge model for a single predator species and either one or two prey species where no predators are present in the prey refuge. They have theoretically illustrated that the ecological factors, competition and refuge, may have a combined destabilizing effect from the evolutionary perspective.

It has often been suggested that refuges are a crucial factor allowing prey to persist with predators, relatively few quantitative studies have directly addressed the patterns or implications of refuge use. But these studies ignore the dynamic nature of refuge use, that is, the variation of populations according to the rates of emergence from and reentry into refuge. According to our best knowledge it is relevant to point out from the above literature survey that no attempt has been made to study the dynamics of a prey predator system which incorporates simultaneous effects of refuge and migration of the population from refuge area to predatory area. In the present paper we, therefore, model the dynamics of a prey–predator system where migration of the prey population is taken into consideration from refuge area to predatory area. To understand natural communities it is important to study the responses of prey to the predators and investigate the mechanisms determining the outcome of interactions involving the predators. The intuitive view of refuges is to stabilize equilibria and analyze the effects predation in predator–prey models. However, including some realistic features, we develop a simple model of refuge use that predicts survival patterns of prey and predator