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Original article

Rapid quantification of uncertainty in permeability and porosity of oil reservoirs for enabling predictive simulation☆

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Abstract

One of the most difficult tasks in subsurface flow simulations is the reliable characterization of properties of the subsurface. A typical situation employs dynamic data integration such as sparse (in space and time) measurements to be matched with simulated responses associated with a set of permeability and porosity fields. Among the challenges found in practice are proper mathematical modeling of the flow, persisting heterogeneity in the porosity and permeability, and the uncertainties inherent in them. In this paper we propose a Bayesian framework Monte Carlo Markov Chain (MCMC) simulation to sample a set of characteristics of the subsurface from the posterior distribution that are conditioned to the production data. This process requires obtaining the simulated responses over many realizations. In reality, this can be a prohibitively expensive endeavor with possibly many proposals rejection, and thus wasting the computational resources. To alleviate it, we employ a two-stage MCMC that includes a screening step of a proposal whose simulated response is obtained via an inexpensive coarse-scale model. A set of numerical examples using a two-phase flow problem in an oil reservoir as a benchmark application is given to illustrate the procedure and its use in predictive simulation.

Keywords: Dynamic data integration; GPU; Two-phase flows; Uncertainty quantification

1. Introduction

Reliable characterization of subsurfaces is one of the most challenging tasks in the flow through porous media community. A common practice is to use the available dynamic data to guide the characterization of a particular reservoir. This practice, which is coined "history matching," amounts to adjusting the reservoir model until it closely reproduces the recorded data. In reality, history matching requires sampling subsurface characteristics, such as porosity and permeability fields, submitting them as input parameters to the reservoir model (usually expressed in terms of a

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set of governing mathematical principles), obtaining the output of simulated responses, and then comparing them with the measured data.

However, direct simulations of flow and transport problems may be computationally expensive or infeasible to compute. Ideally, the level of mesh resolution on which the flow and transport are sought has to be in a comparable scale with the level of subsurface characteristics resolution. Often this would require designing a highly unstructured grid along with the possibility of local refinements in various subregions. Putting this into the scenario of sampling large numbers of subsurface characteristics, one can see the tremendous computational load that has to be committed in history matching.

On a related topic, it is not immediately obvious how to rigorously represent the subsurface characteristics before sending them to the reservoir model. In this regard, the predicament lies on the uncertain nature of these characteristics. In our context, the values of these characteristics have to be projected to the underlying computational mesh on which the simulated responses are to be evaluated. This will translate into the dimensional immensity of the uncertainty space which greatly multiplies the already huge computational cost.

To recapitulate, establishing a complete statistical description of the subsurface that is consistent with measurement data necessitates focusing the effort on addressing three fundamental problems, namely, (1) proper modeling of the flow and transport in a subsurface, (2) appropriate parametrization of the subsurface characteristics in hopes of representing the uncertainty correctly, and (3) accurate numerical procedures that make the overall computational work tractable. Our thesis in this paper is that these fundamental problems can be resolved by implementing careful modeling reduction techniques. The reduced system is then placed as a black-box in the framework of Bayesian statistical inference in combination with the Markov chain Monte Carlo (MCMC) method. This statistical approach aims at generating a Markov chain from which a stationary, posterior distribution of the subsurface characteristics may be constructed. In our case, the goal is to describe the posterior distribution of the permeability and porosity conditioned to known measurement data.

Furthermore, as the uncertainty space of a permeability and porosity field may be exceptionally large, a reduction of the space dimension is a step which must be performed for computationally feasible simulations. The well known Karhunen–Loève expansion [21] allows for parametrization of the random space and thereby leads to the desired reduction. The parametrization is achieved by solving an eigenvalue problem involving a given covariance structure and the assumption of uncorrelated random coefficients. This technique has been used previously in flow through porous media applications (see, e.g. [10,11]), where a random permeability field defined on a large number of underlying grid points is expressed using the expansion.

Many authors studied Bayesian methods for inverse problems in reservoir modeling. We refer the reader to [19,23] for recent works in inverse problem in applications of flow through porous media. Lee et al. [19] used an intrinsically stationary Markov random field, which compares favorably to Gaussian process models and offers some additional flexibility and computational advantages, for the choice of prior for the unknown permeability field. Through a Bayesian approach, using MCMC methods to explore the high-dimensional posterior distribution, they demonstrated how to characterize an aquifer based on flow data. In contrast, Efendiev et al. [6,10,11] used the Karhunen–Loève expansion to parametrize the permeability field.

In practice however, MCMC can be computationally expensive, particularly for solving forward problems in real flow problems. Higdon et al. [17] presented a methodology for improving the speed and efficiency of an MCMC by combining runs on different scales. They introduced a coarse-scale to make the MCMC chain run faster and better explore the posterior, and linked the coarse chain back to the original fine-scale chain of interest. The proposed coupled MCMC runs more efficiently without sacrificing the accuracy achieved at the fine-scale. In [10], a two-stage MCMC, that utilizes inexpensive coarse-scale models to screen out detailed flow and transport simulations, was used to explore the posterior distribution of permeability field. In the first stage, a new proposal is first tested at the coarse-scale model. If the proposal passes the testing at the coarse-scale model, then at the second stage the fine-scale simulation will be run and this fine-scale run is very expensive compared to the coarse-scale run.

We use the Metropolis–Hasting MCMC [16] with a random walk sampler to explore the posterior distribution. To achieve high efficiency in this process, we implement a tuning mechanism so that the sampler has an acceptance rate that is neither too high nor too low. Since we explore a high-dimensional posterior distribution, we implement a component-wise auto-tuning mechanism [14] at the coarse-scale stage in a two-stage MCMC.

It has been generally known that permeability and porosity are two of the most important reservoir properties governing the movement and storage of the fluids. To the best of the authors' knowledge, the current paper is the first