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Original article

Implicit time advancing combined with two finite-volume methods in the simulation of morphodynamic flows

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Abstract

Numerical simulation of morphodynamic problems is considered. The physical model is based on the shallow-water equations coupled with the Exner equation closed by the Grass model to describe the time evolution of the bed profile. The SRNH predictor–corrector scheme and a modified Roe scheme for non-conservative systems of equations are considered for space discretization. Second-order accuracy in space is achieved through variable reconstruction. These schemes were previously used in the simulation of the considered problems together with explicit time advancing. Linearized implicit time-advancing versions are generated here, in which the flux Jacobians are computed through automatic differentiation. Second-order accuracy in time is obtained through a backward differentiation formula associated with a defect-correction approach. For both the considered numerical methods, the explicit and implicit versions are compared in terms of accuracy and efficiency for one-dimensional and two-dimensional morphodynamic problems characterized by different time scales for the evolution of the bed and of the water flow.

Keywords: Shallow water; Implicit time advancing; Defect correction; Exner equation; Morphodynamic flows; Finite-volume schemes

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1. Introduction

Sediment transport processes, caused by the movement of a fluid in contact with the sediment layer, are present in many environmental problems and engineering applications, such as, for instance, severe climate waves, seabed response to dredging procedures, harbor siltation or transport in gravel-bed rivers. Physical models of various complexity can be used to describe the above mentioned phenomena. We consider herein a model constituted by a hydrodynamical part coupled with a morphodynamical component. For the hydrodynamical part the classical shallow-water equations are adopted, which are widely used for scientific and practical studies of this type of problems. The morphological part is described by one single sediment transport equation, i.e. the mass conservation equation, also known as the Exner equation. Sediment load can be taken into account through suitable closures for the sediment transport rates in the Exner equation. Many different models are available in the literature (see e.g. [9] for a review). As a first step, the Grass expression [14] is considered herein, which provides one of the most popular and simple models.

A huge amount of work has been carried out in the last decades to develop numerical methods for the simulation of the previous system of equations (see e.g. the references in [3–5,7,9,11]). Time advancing is usually carried out by explicit schemes. Explicit time-advancing schemes have several positive features; indeed, they are very easy to be implemented and they do not require the solution of large linear systems. On the other hand, it is well know that for explicit time advancing the time step is limited by numerical stability. When the water flow and the bed evolution are not fast, this time step restriction can lead to large computational costs. For these cases, it may be advantageous to use implicit schemes. In a previous work [8], we combined an existing finite-volume method based on the modified Roe scheme for space discretization [9] with implicit time advancing. A practical difficulty with implicit schemes arises from the fact that, in order to avoid the solution of a nonlinear system at each time step, the numerical fluxes must be linearized in time. This is classically done via differentiation by computing the Jacobian of the fluxes with respect to the flow variables. Nevertheless, it is often not convenient to exactly compute the Jacobian matrices, because the computation may be complex for some schemes, as e.g. those involving projector–corrector stages. In order to overcome these difficulties, we used an automatic differentiation tool (Tapenade, [16], http://www-sop.inria.fr/tropics/). A defect-correction approach [19], which consists in iteratively solving linear systems involving the 1st-order flux Jacobians, was finally adopted to obtain 2nd-order accuracy (both in time and space) at limited computational costs. 1D and 2D test cases, characterized by low values of the Froude number and different rates of interaction between the bed and the water flow, were considered in [8]. The effects of an initial irregular shape of the bottom and of a variable inlet height were also investigated in 1D. In all cases, for implicit time advancing the time step was found to be limited by the physical time scale of the considered problem rather than by numerical stability. Whenever the use of large time steps was compatible with the capture of the water flow dynamics and of the bedload evolution, the implicit scheme was found to be far more efficient than its explicit counterpart with a CPU reduction up to more than two orders of magnitude. More precisely, the maximum CFL number reachable with the implicit scheme without any loss of accuracy was observed to be roughly inversely proportional to the constant A_g, determining the velocity of the interaction between the flow and the bed load.

In the present work we use the same methodology, i.e. automatic differentiation and defect correction, to derive a 2nd-order implicit scheme starting from another finite-volume numerical method, the predictor–corrector SRNH scheme, introduced in [1,20] and previously used together with explicit time advancing for sediment transport problems (see, e.g. [2,4,5]). Note that, thanks to the use of automatic differentiation and of defect correction, one of the advantages of the strategy proposed in [8] for constructing implicit schemes from an existing explicit counterpart is indeed that it can be straightforwardly adapted to changes in the spatial numerical discretization or in physical modeling. The specific aim of the present work is to assess to which extent the efficiency gains found in [8] for the implicit version of the MR scheme depend on the spatial discretization method. To this aim implicit and explicit versions of the MR and SRNH schemes, both at 1st and 2nd-orders of accuracy, are applied herein to the same 1D and 2D test case as in [8], characterized by a low Froude and different speeds of interaction between the water flow and the bedload. Moreover, for the different considered values of the parameter A_g in the Grass model, simulations at progressively increasing Froude number are also carried out in 1D, in order to provide a first quantification of how the accuracy limitation on the implicit time step depends on the second main scale characterizing morphodynamic problems, i.e. the one related to the water flow dynamics.