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Numerical analysis for a seawater intrusion problem in a confined aquifer

A. Abudawia, C. Rosier*

Univ Lille Nord de France, F-59000, Lille, France
ULCO, LMPA J. Liouville, BP 699, F-62 228 Calais, France
CNRS FR 2956, France

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Abstract

We simulate a diffuse interface model issuing from a seawater intrusion problem in a confined aquifer. We first use a $P1$ finite element method for which we establish error estimates for any solution sufficiently regular. We propose a finite volume method and we compare the results given by these two methods.

Keywords: Seawater intrusion problem; Finite element method; Finite volume method; Numerical analysis

1. Introduction

Groundwater is a major source of water supply in many parts of the world. In coastal zones, which are densely populated areas, the intensive extraction of freshwater yields to local water table depression causing sea intrusion problems. Then the optimal exploitation of fresh water and the control of seawater intrusion in coastal aquifers are the challenges for the future water supply engineers and managers. We need efficient and accurate models to simulate the transport of salt water front in coastal aquifer. We distinguish two important cases: the case of free aquifer and the one of confined aquifer. In these 2 cases, the aquifer is bounded by two layers, the lower layer is always supposed to be impermeable. For the confined aquifer, the upper surface of the aquifer is impermeable and for the free aquifer, the upper surface is a permeable layer constituted by gravels, sand or alluvia. In this paper, we are interested in efficient numerical algorithms to solve the evolution of the sea front in the case of confined aquifer but we emphasize that we can generalize our result to the case of free aquifer. The basis of the modeling is the mass conservation law for each species (fresh and salt water) coupled with the classical Darcy law for porous media. Of course, freshwater and saltwater are miscible fluids and therefore the zone separating them takes the form of transition zone with variable concentration of salt. But for certain problems, the simulation can be simplified by assuming that each liquid is confined to a well defined portion of the flow domain with an abrupt interface separating the two domains called

* Corresponding author at: ULCO, LMPA J. Liouville, BP 699, F-62 228 Calais, France.
E-mail addresses: abudawia@lmpa.univ-littoral.fr (A. Abudawia), rosier@lmpa.univ-littoral.fr (C. Rosier).

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sharp interface. In this paper, we suppose the existence of a diffuse interface between the fresh and salt water with a constant thickness δ. Using phase function introduced in Allen–Cahn model we obtain a supplementary diffusive term $-\delta\nabla\cdot(\nabla h)$ in the finale equation governing the evolution of the interface depth h. Of course, this term allows us to get more regularity on the solution but also to ensure a maximum principle naturally satisfied by the solution. The second approximation – so called Hydraulic approximation – consists in vertical averaging of the model. We thus assume quasi-horizontal displacements, hence we get a $2D$-vertically averaged model. This assumption is legitimate when the thickness of the aquifer is small compared to the width and length of the aquifer and also when the flow is far from sinks/wells. We refer to [1,7,9], (and a long list of references therein) for more details about sea intrusion problems with sharp interface approach and to [2] about model with constant diffuse interface approach.

The evolution of the depth of the interface and of the freshwater hydraulic head are given by a coupled two-dimensional system consisting of an elliptic and a parabolic equations. In the first part of the paper, we introduce a semi-implicit in time scheme combined with a $P1$ finite element method to discretize our problem. We establish an error estimate for the full discretization. The proof of this result is based on the specific properties of the exact solution, namely the maximum principle satisfied by the solution and a uniform in time bound in L^∞-norm of the gradient of the solution, for sufficiently regular boundary and initial data. In the second part, we present a finite volume scheme based on the intrinsic nature of the model that derive from conservative laws.

The outline of the paper is the following one. Section 2 is devoted to the model and its derivation. In Section 3 all mathematical notations and global in time existence results are stated. Section 4 is devoted to the presentation and error analysis of the finite element scheme. In Section 5, we present the finite volume method. In Section 6, we conducted numerical experiments in order to compare the approximate solutions obtained by these two schemes.

2. Modeling

2.1. Conservation laws

The basis of the modeling is the mass conservation law for each species (fresh and salt water) coupled with the classical Darcy law for porous media. The Darcy law relating together the effective velocity q of the flow and the hydraulic head Φ reads:

$$q = -K grad(\Phi), \quad K = \frac{\kappa\rho g}{\mu}. \tag{1}$$

The hydraulic head is given by

$$\Phi = \frac{P}{\rho g} + z,$$

where ρ and μ are respectively the density and the viscosity of the fluid, κ is the permeability of the soil and g the gravitational acceleration constant.

The matrix K is the hydraulic conductivity. It expresses the ability of the ground to conduct water, K is proportional to κ the permeability of the ground which only depends on the characteristics of the porous medium and not on the fluid.

At this point, introducing specific index for the fresh (f) and salt (s) waters and using (1), we derive from the mass conservation law for each species (fresh and salt water) the following model:

$$\begin{aligned}&S_f\partial_t\Phi_f + \nabla\cdot q_f = Q_f, \quad q_f = -K_f\nabla\Phi_f,\ K_f = kg\rho_f/\mu_f,\\&S_s\partial_t\Phi_s + \nabla\cdot q_s = Q_s, \quad q_s = -K_s\nabla\Phi_s,\ K_s = kg\rho_s/\mu_s.\end{aligned}$$

The coefficient of water storage S_i $(i = f, s)$ characterizes the workable water volume. It accounts for the rock and fluid compressibility. In general, for example in confined aquifer, this coefficient is extremely small (of order of water compressibility). For the free aquifer, it is of order of the porosity of the medium.

Applying the vertical averaging of the model, we integrate the mass conservation law between the upper surface h_1 and the depth of the interface h in the fresh water zone and between the depth of the interface h and the lower surface h_2, in the salt water zone (cf. Fig. 1).