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# On a family of nonlinear cell-average multiresolution schemes for image processing: An experimental study

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**Abstract**

This paper is devoted to a new family of nonlinear cell-average multiresolution schemes and its applications to image processing. The algorithms are based on nonlinear reconstruction operators with several desirable features: the reconstructions are third-order accurate in smooth regions, the data used is always centered with optimal support and they are adapted to the presence of discontinuities.

The goal is to obtain similar properties as linear multiresolution schemes but avoiding the classical Gibbs phenomenon of this type of reconstructions. Applications to image compression and denoising will be presented.


*Keywords:* Cell-average multiresolution; Nonlinear subdivision schemes; $p$-power means; Compression; Denoising

## 1. Introduction

In the last years, various techniques to improve the classical linear multiresolutions of wavelet type have led to nonlinear multiresolutions [10,12,14,15,21,22].

In [2], in the context of image compression, a new nonlinear point-value multiresolution, called PPH (for Piecewise Polynomial Harmonic), has been presented. Convergence and stability of its associated subdivision scheme are derived [14]. In [3], we established the stability of the PPH multiresolution that, due to nonlinearity is not a consequence of the stability of the associated subdivision scheme. Edge resolution, robustness with regard to texture or noise, accuracy and compression capabilities have been numerically investigated.

In most of the considered models in the study of image processing methods, the starting point is to assume that images are $L^1$ functions with certain regularity. For instance, it is possible to find models working in $BV$, $B^1_{1,1}$ or

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$W^{1,1}$, [11,13,16]. Thus, it is natural to consider cell average discretization operators when $L^1$ is the space where the original function lives.

In this paper we introduce a new family of cell-average multiresolution schemes that work in the cell-average framework that is a setting more adapted to image applications. We will make use of the so called *p-power means* [24]. In [5] a particularization of these schemes ($p = 2$) was used for image compression.

In the numerical experiments we consider a particularization of the original family presented, comparing its performance through some numerical examples for color image compression and color image denoising. The aim is to reduce the Gibbs phenomenon [17] of linear multiresolution schemes while maintaining similar performance.

This paper is organized as follows: In Section 2 we recall the Harten framework and we present a new family of nonlinear cell-average multiresolution schemes. In Section 3, we find the exponent $p$ that performs better for the $p$ means. Using the bivariate context of tensor product, the new family of reconstructions is tested in Section 3 on color images, allowing to compare the performances of linear and nonlinear schemes.

## 2. The Harten framework

In this Section we review Harten's framework for multiresolution, considering the cell-average setting.

Harten's general framework for multiresolution [9,18,19] relies on two operators, decimation and prediction, that define the basic interscale relations. These operators act on linear vector spaces, $V^k$, that represent the different resolution levels ($k$ increasing implies more resolution)

$$D_k^{k-1} : V^k \to V^{k-1} \tag{1}$$

$$P_{k-1}^k : V^{k-1} \to V^k \tag{2}$$

and they must satisfy two requirements of algebraic nature: (a) $D_k^{k-1}$ must be a linear operator and (b) $D_k^{k-1} P_{k-1}^k = I_{V^{k-1}}$ (consistency), i.e., the identity operator on the lower resolution level represented by $V^{k-1}$.

### 2.1. *The cell-average multiresolution setting*

Let us consider a set of nested grids in $\mathbb{R}$:

$$X^k = \{x_j^k\}_{j\in\mathbb{Z}}, \qquad x_j^k = jh_k, \quad h_k = 2^{-k}, \ k = 0, \dots, L,$$

where we consider the discretization

$$\mathcal{D}_k : L^1(\mathbb{R}) \to V^k, \qquad f_j^k = (\mathcal{D}_k f)_j = \frac{1}{h_k} \int_{x_{j-1}^k}^{x_j^k} f(x)dx, \quad j \in \mathbb{Z}, \tag{3}$$

where $L^1(\mathbb{R})$ is the space of absolutely integrable functions in $\mathbb{R}$ and $V^k$ is the space of sequences at resolution $k$.

From the additivity of the integral, we obtain the decimation steps:

$$f_j^{k-1} = (D_k^{k-1} f^k)_j = \frac{1}{h_{k-1}} \int_{x_{j-1}^{k-1}}^{x_j^{k-1}} f(x)dx = \frac{1}{2h_k} \int_{x_{2j-2}^k}^{x_{2j}^k} f(x)dx = \frac{1}{2}(f_{2j-1}^k + f_{2j}^k).$$

The consistency requirement for $P_{k-1}^k$ becomes

$$f_j^{k-1} = (D_k^{k-1} P_{k-1}^k f^{k-1})_j = \frac{1}{2}((P_{k-1}^k f^{k-1})_{2j-1} + (P_{k-1}^k f^{k-1})_{2j}).$$

Hence, if $f^{k-1} = D_k^{k-1} f^k$, then the two last equations imply that the prediction errors satisfy

$$e_{2j-1}^k = f_{2j-1}^k - (P_{k-1}^k f^{k-1})_{2j-1} = (P_{k-1}^k f^{k-1})_{2j} - f_{2j}^k = -e_{2j}^k,$$

which shows the redundancy inherent in the prediction error.