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Clifford Fourier–Mellin moments and their invariants for color image recognition

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Abstract

The aim of this paper is to propose a Fourier–Mellin moments transform, for color object recognition, based on the Clifford algebra valued functions over $\mathbb{R}_+^* \times \mathbb{S}^1$. We use these moments to construct descriptors that are invariant under rotation, scale and translation transformations. Application of these invariants in color image recognition is emphasized. Experiment results demonstrate the advantage of the proposed method compared to existing ones.

Keywords: Clifford algebra; Fourier–Mellin moments; Invariants; Color image recognition

1. Introduction

Moment methods are employed in image analysis and are used in objects recognition, but need to be invariants under geometrical transformations. The first significant paper discussing the use of image moment invariants for two dimensional pattern recognition was published by Hu [6]. However, the general problems to be considered are that of processing three dimensional image directly without losing color information and achieving rotation, scale and translation invariant object recognition. For this later, the Fourier–Mellin moments are a good tool, these moments were first introduced in [2,8]. Many definitions for such moments using quaternions or Clifford algebras were introduced in the literature. Recently, Guo and Zhu defined in [4] Fourier–Mellin transform moments for color images, their approach is based on the use of the quaternion algebra. Since quaternions can be represented by geometric algebra, we propose, in this paper, a new definition of Fourier–Mellin moments based on the Clifford algebra (ClFMMs), using the traditional Fourier–Mellin transform and the Clifford Fourier transform introduced by Batard [1]. Related to this context, a generalization in the Clifford algebras $Cl(p,q)$, $p+q=2$ is proposed also by Hitzer in [5], the author uses a set of two real square roots of -1, $f, g \in Cl(p,q)$, $f^2 = g^2 = -1$.

We recall in Section 2 the usual definition of the Fourier transform using group actions, the Fourier–Mellin transform and the Clifford Fourier transform. Then we introduce the Clifford Fourier–Mellin moments in Section 3 and

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we propose its application to color image recognition in Section 4. In Section 5, we give a conclusion and perspectives for future research.

2. Preliminaries

2.1. Fourier transform and group actions

Let G be a locally compact abelian group and $\hat{G}$ its dual (the set of all irreducible and unitary representations of G, that are in this case one-dimensional and called characters). The Fourier transform of a function $f \in L^2(G; \mathbb{C})$ is defined on $\hat{G}$ by

$$\hat{f}(\lambda) = \int_G f(x)\varphi_\lambda(x^{-1})\, d\nu(x), \tag{1}$$

where ν is the Haar measure of G and φ is an irreducible and unitary representation of G. For more details see [3]. Applying this formula to the cases:

1. $G = \mathbb{R}^m$ then $\hat{G} = \mathbb{R}^m$ and $\varphi_\lambda(x) = \mathrm{e}^{i\langle\lambda, x\rangle}$
$$\hat{f}(\lambda) = \int_{\mathbb{R}^m} f(x)e^{-i\langle\lambda,x\rangle}\, dx,$$
is a multi-dimensional Fourier transform.
2. $G = \mathbb{S}^1$ then $\hat{G} = \mathbb{Z}$ and $\varphi_n(x) = e^{inx}$
$$\hat{f}(n) = \int_{\mathbb{S}^1} f(\theta)e^{-in\theta}\, d\theta,$$
is the Fourier coefficient for a 2π periodic function.
3. $G = \mathbb{R}_+^*$ then $\hat{G} = \mathbb{R}$ and $\varphi_\alpha(x) = x^{i\alpha}$
$$\hat{f}(\lambda) = \int_{\mathbb{R}_+^*} f(x)x^{-i\alpha}\frac{d(x)}{x} = \int_{\mathbb{R}_+^*} f(x)x^{-i\alpha-1}\, dx,$$
is the Mellin transform.

If $\check{f}$ belongs to $L^1(\hat{G}, \nu_{\hat{G}})$ then the inverse Fourier transform exists and it is defined by

$$\check{f}(x) = \int_G \hat{f}(\lambda)\varphi_\lambda(x)\, d\nu_{\hat{G}}(\lambda) \tag{2}$$

where $\nu_{\hat{G}}$ is the Plancherel measure of $\hat{G}$ and φ_λ is an irreducible and unitary representation of G.

2.2. Fourier–Mellin moments

The Fourier–Mellin transform is expressed in polar coordinates (r, θ), where $r > 0$ and $\theta \in \mathbb{S}^1$ ($\mathbb{S}^1$ denotes the unit circle in the plane $\mathbb{R}^2$) with the multiplication defined as $\{(r, \theta) \cdot (r', \theta') = (r \cdot r', \theta + \theta')\}$.

The direct similarity group $\mathbb{R}_+^* \times \mathbb{S}^1$ (which is equivalent to the space of polar coordinates) is a locally compact and abelian group. The Haar measure is $d\nu(r, \theta) = \frac{dr}{r}d\theta$ and the dual group of $\mathbb{R}_+^* \times \mathbb{S}^1$ is $\mathbb{R} \times \mathbb{Z}$. Hence, the Fourier–Mellin transform is defined as follows

$$\hat{f}(k, v) = \int_0^{+\infty}\int_0^{2\pi} f(r, \theta)r^{-iv}e^{-ik\theta}\frac{dr}{r}d\theta. \tag{3}$$

Let f be a function belonging to $L^2(\mathbb{R}_+^* \times \mathbb{S}^1; \mathbb{C})$, the Fourier–Mellin moments of f are defined as follows

$$M_{k,v}(f) = \int_0^{+\infty}\int_0^{2\pi} f(r, \theta)r^{v-1}e^{-ik\theta}\, dr d\theta. \tag{4}$$

2.3. Clifford Fourier transform

A Clifford Fourier transform was proposed by Batard [1] using group actions. The idea was to generalize the usual definition based on the characters of abelian groups, by considering group morphisms from $\mathbb{R}^2$ to Spin(4). These