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Mass-conserving cavitation model for dynamical lubrication problems. Part I: Mathematical analysis

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Abstract

In this paper we prove an existence result for a variety of rotor-bearing systems, namely journal bearing, piston ring–liner and mechanical seal systems. These results are shown for a fixed geometry for different boundary conditions. The mathematical model considered is the mass-conserving Elrod–Adams in presence of cavitation and in the unsteady case. Also a local existence of a dynamical problem is given.

Keywords: Lubrication; Dynamical system; Cavitation; Elrod and Adams' model; Parabolic partial differential equation

1. Introduction and setting problem

In the design of machines with rotating or alternating motion (e.g., journal bearings, piston rings, mechanical seals), the bearing dynamics is essential to predict the machine behavior under different operating conditions (e.g., different rotor speeds, applied loads). Therefore, the availability of an accurate mathematical model of the equations of motion is very important. The complete equations of motion require a model for the hydrodynamic forces acting on the journal in case of journal bearing system, on the ring in case of piston-ring–liner system and on the rotor in case of mechanical seal system. These forces are dependent on the pressure field within the bearing clearance which, in turn, is governed by the Reynolds equation [10] which reads, in unsteady case,

$$\nabla \cdot (h^3 \nabla p) = \nabla \cdot (\boldsymbol{V} h) \tag{1}$$

where h is the distance (denoted clearance or film thickness) between the two surfaces of the lubricated system, $\boldsymbol{V}$ is the relative velocity and p the fluid film pressure. This equation calculates the fluid pressure given the film thickness, squeeze velocity, and the pressure at the boundaries. Let us remark that this model neglects the fluid cavitation.

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A significant body of mathematical studies of such problem can be found in the literature (see, e.g., [2,3]).

Cavitation in a liquid can be defined as the formation of pockets of gas due to the liquids' inability to sustain large sub-ambient pressures [9]. This condition is often encountered in machine elements in relative motion that are separated by a lubricant film [4] (e.g. journal bearings, sliders, piston ring–liner conjunction and mechanical seals). In the piston ring–liner conjunction, cavitation is the results of sudden lubricant pressure drop in the diverging cross section of the ring [11]. This leads to transition of the fluid from liquid to gas–liquid mixture.

In order to take into account the cavitation phenomenon, a modification is made in the Reynolds equation by introducing a new unknown θ (the fluid fraction variable which takes values between zero and one). This model is called the mass-conserving Elrod–Adams model and it is by now commonly accepted as a plausible concept and easy-to-implement tool for simulation in hydrodynamic lubrication involving cavitation.

Denoting by $\Omega \subset \mathbb{R}^2$ the region where the two surfaces are in proximity, let Ω^+ be the pressurized region of the bearing ($p > 0$) where the film is complete ($\theta = 1$). The cavitated region (incomplete film, i.e., $\theta < 1$ and $p = 0$) is denoted by Ω_0. At the internal boundary Σ between these two regions (cavitation boundary), which is an unknown of the problem, the mathematical conditions are the continuity of the pressure (p tends to zero when approaching Σ from either side) and the conservation of the mass of lubricant. Notice that Ω^+, Ω_0 and thus Σ, change in time.

With these definitions, the well-known mass-conserving mathematical model (in the $p - \theta$ form proposed by Elrod and Adams [8]) reads

$$\nabla \cdot (h^3 \nabla p) = \nabla \cdot (\boldsymbol{V} \theta\, h) + 2\, \frac{\partial(\theta h)}{\partial t} \quad \text{in } (\Omega \setminus \Sigma) \times]0, T[\tag{2}$$

$$p > 0,\ \theta = 1 \quad \text{in } \Omega^+(t) \times]0, T[\tag{3}$$

$$p = 0,\ \theta < 1 \quad \text{in } \Omega_0(t) \times]0, T[\tag{4}$$

$$p = 0 \quad \text{on } \Sigma(t) \tag{5}$$

supplemented with the mass-conservation condition at the cavitation boundary

$$(h_0\, \theta_0 - h_+)\ \boldsymbol{V} \cdot \hat{n} + h_+^3 \left(\frac{\partial p}{\partial n}\right)_+ = 2\,(h_0\, \theta_0 - h_+)\ q_n \quad \text{on } \Sigma(t) \tag{6}$$

and the initial condition

$$\theta(x_1, x_2, t = 0) = \theta_I(x_1, x_2), \quad (x_1, x_2) \in \Omega \tag{7}$$

where

- $\boldsymbol{V} = \boldsymbol{V}(x_1, x_2, t)$ is the non-dimensional sliding velocity, assumed depending in time and $x = (x_1, x_2)$,
- $\hat{n}$ is the unit vector normal to Σ, oriented outwards from Ω^+,
- q_n represents the local normal velocity at which Σ is moving,
- the subscripts 0 and + refer to the limit values of the variables as Σ is approached from the cavitated and active regions, respectively,
- $\theta_I \in L^\infty(\Omega)$ and $0 \le \theta_I \le 1$ a.e in Ω.

The expression of the gap thickness $h(x_1, x_2, t)$ depends on the geometry of the mechanical system considered and is given in (10) and (13) for each of the studied systems.

Obviously we must add to previous equations the boundary data on p. This depends on the bearing system considered. In this work we consider two machines: (i) journal bearing (Fig. 1), (ii) piston-ring–liner (Fig. 2). The domain Ω and boundary conditions are as follows:

(i) **Journal bearing**: Fig. 1 displays a scheme of the journal bearing (Fig. 1(a)) together with the associated coordinate system, computational domain (Fig. 1(b)), some dimensions and boundary conditions. The domain considered here is $\Omega =]0, 2\pi[\times]0, L[$ with L the length of the bearing. In the present case we consider the following boundary conditions