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Mass-conserving cavitation model for dynamical lubrication problems. Part II: Numerical analysis

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Abstract

Numerical results are presented for three fully dynamical lubrication devices (piston ring–liner, journal bearing and mechanical seals) based on the Elrod–Adams model for the Reynolds equation and Newmark scheme for the motion equations. For each lubricated system we give the mathematical model, the corresponding non-dimensional problem and numerical results. We give also the full algorithm in the case of the piston-ring–liner system.

Keywords: Lubrication; Dynamical system; Cavitation; Elrod and Adams' model; Numerical simulations

1. Introduction

In the design of machines with rotating or alternating motion (e.g., journal bearings, piston rings, mechanical seals), the bearing dynamics is essential to predict the machine behavior under different operating conditions (e.g., different rotor speeds, applied loads). Therefore, the availability of an accurate mathematical model of the equations of motion is very important. The complete equations of motion require a model for the hydrodynamic forces acting on the journal in case of journal bearing system, on the ring in case of piston-ring–liner system and on the rotor in case of mechanical seal system. These forces are dependent on the pressure field within the bearing clearance which, in turn, is governed by the Elrod and Adams model [6].

The rotor-bearing system equations are highly non-linear and, in general, require considerable numerical effort to be solved.

Though many algorithms have been published (see Refs. [10,12,17]), the codes implementing them treat only the Reynolds equation rather than the complete dynamical problem. The numerical algorithm of the mass-conserving lubrication model with the $p-\theta$ formulation (p is the film pressure and θ the saturation of the fluid) of Elrod–Adams [6]

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for dynamically-loaded bearing is given in [1] where we presented a numerical algorithm for a journal bearing with periodic boundary conditions.

In this paper, we give numerical results in case of three mechanical systems: (i) piston ring–liner; (ii) journal bearing with a given flux and (iii) mechanical seals.

We decided to consider these three mechanical problems for the following reasons.

- The first one stands from an educational point of view. Indeed we do not find in the mathematical literature related to lubrication, works explaining the various problems and their interest in the practice.
- The second reason stands from a numerical point of view. By considering the dynamical systems of the three problems, the numerical results are completely different and therefore it is interesting to give an idea of the pressure profile and the dynamical behavior of each mechanical system.
- Another reason is the kind of normalized equation that is necessary to consider for each case; this is detailed in this work.

For a fixed geometry we have shown in [4] an existence result of the unsteady Elrod–Adams problem for the three mechanical systems. We have also given in [4] a local time existence of the corresponding dynamical problems. This work is considered as an accompanying numerical paper.

The numerical treatment is that described in [1] with particular adaptation to the different cases (piston ring–liner, journal bearing with given flux and mechanical seal systems). It consists of a finite volume, conservative method with upwinding discretization of the Couette flux ($\nabla \cdot (\mathbf{V}\, h\theta)$) and centered discretization of the Poiseuille flux ($\nabla \cdot (h^3 \nabla p)$) and an iterative imposition of the cavitation conditions ($p \geq 0$; $\theta < 1 \Rightarrow p = 0$; $p > 0 \Rightarrow \theta = 1$) by means of a Gauss–Seidel-type algorithm. The motion equation is discretized by a Newmark scheme, which is built into the overall iterative process.

The complete algorithm is given in Section 2.2 for the piston ring–liner system. In the case of journal bearing the code is the same as in [1] with an adaptation for the boundary conditions (24)–(25). For the mechanical seal system, the code is completely rewritten because the Reynolds equation is different due to a change of variables (see Eq. (36)).

The paper is organized as follows. In Section 2 we present the piston ring–liner system and give the mathematical model, the corresponding nondimensional problem, the numerical algorithm and some numerical results. Section 3 is devoted to the journal bearing device, giving the mathematical model, the corresponding nondimensional problem and some numerical results. In Section 4 we give similar results for mechanical seals systems. We end this paper by a conclusion.

2. Piston ring

Pistons are present in numerous mechanical applications. The most common one is the internal combustion engine, in particular in the automotive industry. The vertical movement of the piston, which takes place inside a cylinder, accommodates a variation of volume of the combustion chamber situated above the piston. The piston movement allows for the transformation of the chemical energy released during the combustion of the fuel into mechanical energy. The piston admits three ring segments of circular shape in general, which form a dynamic joint between the fuel-fired room and the driving crankcase. The segments (rings) play two roles; (i) they prevent the passage of gases from the chamber to the crankcase, and (ii) they limit the consumption of motor oil in the fuel-fired chamber while guaranteeing a correct lubrication of the contact. Comprehensive theoretical and numerical modeling of piston ring lubrication has been undertaken in [3,14–16].

We can see a scheme of the ring–liner contact in Fig. 1 and a corresponding computational domain in Fig. 2. The zone of full film fluid (the pressure of the fluid is strictly positive) is noted by Ω_+ and the cavitation zone (the pressure of the fluid is zero) is noted by Ω_0.

For the sake of simplicity we consider an idealized parabolic piston ring at a constant speed over cylinder wall with one degree of freedom, the vertical translation. The dynamics of the ring is governed by the forces acting on it along the x_3-direction. These forces are described below:

The *load* imposed on the ring–liner bearing comes from the elastic response of the ring. It points outwards (i.e.; along $-x_3$) and is assumed constant. We denote by W_a its value *per unit length along* x_2.

The *hydrodynamic force* originates from the pressure $p(x_1, x_2, t)$ that develops in the oil film between the ring and the liner. Its value per unit length along x_2 is given by